Tomography
With Deep Learning

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August 8, 2019
Outline

- Deep Reason
- Experimental Results
- Explainable AI
- Promising Topics
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- Deep Reason
- Experimental Results
- Explainable AI
- Promising Topics
Industrial vs Info/Intelligence Techniques
Pre-cursor for Data-driven Recon (2012)

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Low-Dose X-ray CT Reconstruction via Dictionary Learning

Qiong Xu, Hengyong Yu, Senior Member, IEEE, Xuanqin Mou, Lei Zhang, Member, IEEE, Jiang Hsieh, Senior Member, IEEE, and Ge Wang, Fellow, IEEE
Data-driven Radiomics (Yu & Wang, 2013)

Figure 1. Flowchart of the proposed tensor dictionary and neural network analysis for lung cancer low-dose CT screen to differentiate true/false positive/negative results.
X-Ray Imaging Innovations for Biomedicine

Friday, February 12, 2016: 3:00 PM-4:30 PM
Coolidge (Marriott Wardman Park)

In computed tomography (i.e., CT scans), X-rays generated in an emission source are used to illuminate an organism, project shadows, and undergo measurement in a detector array. Spatiotemporal multiplexing of X-ray shadows enables computational synthesis of people’s internal structures. Today, X-ray CT has a central role in clinical imaging, often as the first and only imaging study before definitive intervention for a wide variety of conditions. More than 100 million CT scans are performed worldwide each year. However, current X-ray CT technology is often insufficient to differentiate benign and malignant etiologies, describe tissue tumor types and grades, or predict early response to therapy. X-ray CT involves ionizing radiation, which has drawn concerns over potential risk of induced cancer formation. This symposium highlights recent improvements in X-ray sources, detectors, and reconstruction algorithms that promise to address some of these long-standing challenges. For example, photon-counting detectors add a spectral dimension to the information content, X-ray gratings extract refractive and elastic scattering features that improve soft tissue contrast, and contemporary reconstruction methods refine image quality with reduced radiation dose. New CT scanners are being developed to offer superior imaging performance and minimize production, deployment, and operation costs. CT scanners also serve as a source of big data that can be archived on the cloud and reused for smarter imaging and universal accessibility.

Organizer: Ge Wang, Rensselaer Polytechnic Institute
Co-Organizer: Mannudeep Kaira, Massachusetts General Hospital
AI Talk at AAAS

The Technology of Artificial Intelligence

Saturday, February 13, 2016: 3:00 PM-4:30 PM
Marshall Ballroom North (Marriott Wardman Park)

Demis Hassabis, DeepMind, London, United Kingdom

Dr. Demis Hassabis is the Co-Founder and CEO of DeepMind, the world’s leading General Artificial Intelligence (AI) company, which was acquired by Google in 2014 in their largest ever European acquisition. Demis will draw on his eclectic experiences as an AI researcher, neuroscientist and videogames designer to discuss what is happening at the cutting edge of AI research, its future impact on fields such as science and healthcare, and how developing AI may help us better understand the human mind.
Roadmap for Deep Recon

A Perspective on Deep Imaging

GE WANG (Fellow, IEEE)
Department of Biomedical Engineering, Biomedical Imaging Center, Center for Biotechnology and Interdisciplinary Studies, Rensselaer Polytechnic Institute, Troy, NY 12180, USA
Corresponding author: G. Wang (ge-wang@ieee.org)

ABSTRACT The combination of tomographic imaging and deep learning, or machine learning in general, promises to empower not only image analysis but also image reconstruction. The latter aspect is considered in this perspective article with an emphasis on medical imaging to develop a new generation of image reconstruction theories and techniques. This direction might lead to intelligent utilization of domain knowledge from big data, innovative approaches for image reconstruction, and superior performance in clinical and preclinical applications. To realize the full impact of machine learning for tomographic imaging, major theoretical, technical and translational efforts are immediately needed.
ACKNOWLEDGMENT

The author is grateful for inspiring discussions with Drs. James Brink (Harvard MGH), Wenxiang Cong (RPI), Juergen Hahn (RPI), Michael Insana (UIUC), Nadeem Ishaque (GE GRC), Mannudeep K. Kalra (Harvard MGH), Xuanqin Mou (Xi’an Jiaotong Univ., China), Jiantao Pu (Univ. of Pittsburg), Dinggang Shen (Univ. of North Carolina), Michael Vannier (Univ. of Chicago), Yan Xi (RPI), George Xu (RPI), Pingkun Yan (Phillips), Hengyong Yu (Univ. of Massachusetts Lowell), Junping Zhang (Fudan Univ., China), Yi Zhang (Sichuan University), and other colleagues who made valuable comments. He also acknowledges the advice from an anonymous reviewer who gave advice on the theoretical limitations of compressed sensing. Figures 7, 8 and 9 were produced by Mr. Qingsong Yang (RPI).
FIGURE 2. Big picture of deep imaging – A full fusion of medical imaging and deep learning. A high likelihood is that the direct paths from data to features and actions may need an intermediate layer essentially equivalent to a reconstructed/processed image.
Image Reconstruction Is a New Frontier of Machine Learning

Ge Wang®, Fellow, IEEE, Jong Chu Ye®, Senior Member, IEEE, Klaus Mueller®, Senior Member, IEEE, and Jeffrey A. Fessler®, Fellow, IEEE

I. INTRODUCTION

OVER the past several years, machine learning, or more generally artificial intelligence, has generated overwhelming research interest and attracted unprecedented public attention. As tomographic imaging researchers, we share the excitement from our imaging perspective [item 1] in the Appendix, and organized this special issue dedicated to the theme of “Machine learning for image reconstruction.” This special issue is a sister issue of the special issue published in May 2016 of this journal with the theme “Deep learning in medical imaging” [item 2] in the Appendix. While the previous special issue targeted medical image processing/analysis, this special issue focuses on data-driven tomographic reconstruction. These two special issues are highly complementary, since image reconstruction and image analysis are two of the main pillars for medical imaging. Together we cover the whole workflow of medical imaging: from tomographic raw data/features to reconstructed images and then extracted diagnostic features/reading.
Strong Momentum of AI/ML

Figure 5: Web of Knowledge search, with “deep learning”, “medical”, and “imaging” all as the topic terms (Left), and with “deep learning” in the article title (Right). Data collected on July 11, 2019.
Major Players of AI/ML

Showing 46,802 records for TOPIC: (deep learning)
A learning revolution

The groundwork for machine learning was laid down in the middle of last century. But increasingly powerful computers – harnessed to algorithms refined over the past decade – are driving an acceleration of applications in everything from medical physics to materials, as Marcus Stephens discovers.

When your bank calls to ask about a suspiciously large purchase made on your debit card at a strange store, it’s unlikely that a brief member of stuff has personally been searching through your account. Instead, it’s more likely that a machine has learned what sorts of behaviour to associate with criminal activity – and that it’s plugged searching unprompted on your account. Quickly and efficiently, a bank’s computers analyse every transaction to look for commonalities in what they think are likely to incriminate you. Any deviation from the norm could lead to an alert.

Machine learning can be applied to any task that involves learning from data, which is why it is widely applied in finance. It’s being applied in many fields, from healthcare and transport to the criminal justice system. Indeed, Ge Wang – a computational biologist from the Foundation for Biomedical Research in the US who is one of those

A new IOP Publishing ebook Machine Learning for Tomographic Imaging by Ge Wang, Yi Zhang, Xiaojing Ye and Xuanqin Mou will be published later this year.

https://physicsworld.com/a/a-machine-learning-revolution
AI/ML Tomography Book from IOP Publishing

Image Analysis, One of Successful Applications of Artificial Intelligence & Machine Learning

Theme of This Book: Tomographic Image Reconstruction

Coauthors of the Book:
Yi Zhang, Xiaojing Yu, Xuanqin Mou

Tomographic Data Acquisition

Data As Tomographic Features

Reconstructed Image
Outline of Our Book

Part I
Basic
Chapter 1
Background Knowledge
Imaging Principles, Prior Info, Vision System, Sparse Coding

Part II
CT
Chapter 4
X-ray Computed Tomography
Data Acquisition, Analytic & Iterative Recon, Scanner

Chapter 5
Deep CT Reconstruction
Image & Data Domain, Hybrid, Unrolled/Inserted, Direct Networks

Part III
MRI
Chapter 6
Magnetic Resonance Imaging
MRI Physics, CS-based Recon, Parallel MRI

Chapter 7
Deep MRI Reconstruction

Part IV
Others
Chapter 9
Image Quality Assessment
General, Specific, Task-based Metrics, Network-based Observers

Chapter 10
Quantum Computing
Wave-Particle Duality, Quantum Gates, Quantum Algorithms

Part V
Appendices
Appendix A
Mathematical Basics
Optimization, Inference, Information Theory

Appendix B
Hands-on Experience
Basic Networks, Deep CT & MRI Networks
Table 1: Three types of tomographic image reconstruction algorithms in an over-simplified comparison (penalization of image reconstruction and topology of network architecture can be complicated.

<table>
<thead>
<tr>
<th>Category</th>
<th>Form</th>
<th>Knowledge</th>
<th>Input</th>
<th>Quality</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytic Recon</td>
<td>( f = O[p] )</td>
<td>Idealized Model, Without Noise</td>
<td>High SNR, Compute</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Iterative Recon</td>
<td>( f^{(k)} = O[p, f^{(k-1)}] )</td>
<td>Physical Model, Image Prior</td>
<td>Low in Various Ways</td>
<td>Decent</td>
<td>Low</td>
</tr>
<tr>
<td>Deep Recon</td>
<td>( f = O_{\alpha_1} \ldots O_{\alpha_0} [p] )</td>
<td>Model, Prior, Big Training Data</td>
<td>Poor, Incomplete</td>
<td>Superior, Task-specific</td>
<td>High</td>
</tr>
</tbody>
</table>
Image Reconstruction: From Sparsity to Data-adaptive Methods and Machine Learning

Saiprasad Ravishankar, Member, IEEE, Jong Chul Ye, Senior Member, IEEE, and Jeffrey A. Fessler, Fellow, IEEE

Abstract—The field of medical image reconstruction has seen roughly four types of methods. The first type tended to be analytical methods, such as filtered back-projection (FBP) for X-ray computed tomography (CT) and the inverse Fourier transform for magnetic resonance imaging (MRI), based on simple mathematical models for the imaging systems. These methods are typically fast, but have suboptimal properties such as poor resolution-noise trade-off for CT. A second type is iterative reconstruction methods based on more complete models for the imaging system physics and, where appropriate, models for the sensor statistics. These iterative methods improved image quality by reducing noise and artifacts. The FDA-approved methods among these have been based on relatively simple regularization models. A third type of methods has been designed to accommodate modified data acquisition methods, such as reduced sampling in MRI and CT to reduce scan time or radiation dose. These methods typically involve mathematical image models involving assumptions such as sparsity or low-rank. A fourth type of methods replaces mathematically designed models of signals and systems with data-driven or adaptive models inspired by the field of machine learning. This paper reviews the progress in medical image reconstruction methods with focus on the two most recent trends: methods based on sparsity or low-rank models, and data-driven methods based on machine learning techniques.

acquisition time) or low-dose or sparse-view data in CT (reducing patient radiation exposure) has been a popular area of research and holds high value in improving clinical throughput and patient experience. This paper reviews some of the major recent advances in the field of image reconstruction, focusing on methods that use sparsity, low-rankness, and machine learning. We focus partly on PET, SPECT, CT, and MRI examples, but the general methods can be useful for other modalities, both medical and non-medical. Other papers in this issue emphasize other modalities.

A. Types of Image Reconstruction Methods

Image reconstruction methods have undergone significant advances over the past few decades, with different paths for various modalities. These advances can be broadly grouped in four categories of methods. The first category consists of analytical and algebraic methods. These methods include the classical filtered back-projection (FBP) methods for X-ray CT (e.g., Feldkamp-Davis-Kress or FDK method [1]) and the inverse Fast Fourier transform and extensions such as
Progress Through Questioning

- **Analytic Reconstruction**
  Given a finite number of projections, the tomographic reconstruction is not uniquely determined (ghosts).

- **Statistical Reconstruction**
  A reconstructed image is strongly influenced by the penalty term, and what you see is what you want to see!

- **Compressed Sensing**
  There is a chance that a sparse solution is not the truth. For example, physiological texture and/or pathological plaques incorrectly eliminated

- **Machine Learning**
  No Maxwell equations for machine learning, and a neural network as a black box is trained to work with big data through parameter adjustment
In Principle, Machine Learning (ML) Can Outperform Analytic Reconstruction (AR), Iterative Reconstruction (IR) / Compressed Sensing (CS)

AR/IR/CS Used as

- **Component** (Such as in the “LEARN” Network)
- **Baseline** (Such as for Image Denoising)
- **IR/CS Enhanced/Replaced by Neural Networks** (As Extensive Priors & Powerful Non-learning Mapping, Driven by Big Data)
Superiority Principle

Classic Result → Neural Network → Superior Image Quality → Neural Network

Classic Algorithm → Neural Network

Big Data
Can New Dog be Better in Old Tricks
Spiral Single-slice CT
“For a given X-ray dose, helical CT allows substantially better longitudinal resolution than conventional CT due to its inherent retrospective reconstruction capability.”

Wang and Vannier
Medical Physics 21:429-433, 1994

Wang, Brink, Vannier
Medical Physics 21:753-754, 1994
Incremental (Left) vs Spiral (Right) Scans Define Imaging Planes Differently. The Former Specifies Imaging Planes Physically/Prospectively, while the Latter Does so Computationally/Retrospectively.
Superior Detectability

Direct Reconstruction

Retrospective Reconstruction

Retrospective Reconstruction Gives Better Lesion Detectability If There Are Sufficiently Many Slices Reconstructed!
Competitive performance of a modularized deep neural network compared to commercial algorithms for low-dose CT image reconstruction

Hongming Shan, Atul Padole, Fatemeh Homayounieh, Uwe Kruger, Ruhani Doda Khera, Chayanin Nitiwarangkul, Mannudeep K. Kalra & Ge Wang

IR Methods vs MAP DL for Low-dose CT

Commercial Iterative Recon (IR) Algorithms in This Study

Our MAP Network-based Deep Learning (DL) with Optimized Depth
Best versus Best

Best Deep Recon (DR) vs Best Iterative Recon (IR) Algorithms in This Study across Vendors, Body Regions, & Readers
Outline

- Deep Reason
- Experimental Results
- Explainable AI
- Promising Topics
Low-dose CT Denoising: FBP + Network

GE Medical CT Denoising (RSNA’18)
Super-resolution for Bone CT

Human Distal Tibia Dataset:
- Low Resolution CT: Siemens FLASH
- Super-resolution CT: GAN-O
- High Resolution CT: Siemens FORCE

With Univ. of Iowa, Dr. Saha’s Group
Ensemble Learning for MRI Super-resolution

MRI Super-Resolution with Ensemble Learning and Complementary Priors

Qing Lyu, Hongming Shan, Ge Wang
Sparse-data CT De-artifacts: “LEARN”

LEARN: Learned Experts’ Assessment-based Reconstruction Network for Sparse-data CT. IEEE Trans. Medical Imaging, June 2018

Fig. 1. Architecture of iCT-Net. The proposed deep neural network consists of a total of 12 layers (L1-L12). The L11 layer is a frozen layer, which means that parameters in this layer are not updated in the training process. Both linear and nonlinear activations are used as indicated in the graphics. $S_\chi$ is a hard thresholding activation function defined in Eq. (2).

Exterior Tomography

Metal Artifact Reduction in CT: Where Are We After Four Decades?

LARS GJESTEBY¹, BRUNO DE MAN², YANNAN JIN², HARALD PAGANETTI³, JOOST VERBURG³, DROSOUA GIANTSOUĐI³, AND GE WANG¹ (Fellow, IEEE)

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Corresponding authors: B. De Man (deman@ge.com) and G. Wang (ge-wang@ieee.org)
A female with diffused subarachnoid hemorrhage (in the red square). CT angiography demonstrated a left middle cerebral artery aneurysm, which was clipped. The display window is [-100 200] HU.

Dual-stream Data Processing Flowchart

Phantom derived from clinical images with metal added

Image with major artifacts

FBP

NMAR Algorithm

Guided filtering

CNN Training

Improved image; minor artifacts remain

CNN learns to correct leftover artifacts

CNN Target

(CNN Input)

Detail image derived from filtering
Interior Tomography

Left: The conventional FBP reconstruction from a complete dataset of a sheep chest CT scan (the white circle identifies our selected ROI).
Right: Our TV-minimization-based interior reconstruction from truncated projections associated with x-rays only through the ROI. The sheep scan was done by Dr. Eric Hoffman, University of Iowa, Iowa City, USA

Fig. 10. The iCT-Net reconstruction results of real human subject data acquired in an abdomen-pelvis scan protocol with the short-scan angular range, super-short scan angular range and interior problem. Dense view reconstruction results are presented in the 2nd column and sparse view reconstruction results are presented in the 3rd column. The corresponding reference images were generated by applying a standard FBP method with a Ram-Lak filter at full FOV ($\varnothing = 50$ cm) from 644 view angles densely sampled across a short-scan angular range. Note that the central portion of the FBP reconstruction without truncation was cropped to generate the reference image for the interior problem with the truncated FOV ($\varnothing = 12.5$ cm).
FBP2ADMIRE: Computational Acceleration

Low-dose Sinogram

Iterative Recon (ADMIRE)

FBP

Deep Learning (CNN)

High Quality Image

Yanbo Zhang, Robert MacDougall and Hengyong Yu; Convolutional neural network based CT image post-processing from FBP to ADMIRE. Proceedings of the 5th CT Meeting, pp.411-414, 2018
Outline

- Deep Reason
- Experimental Results
- Explainable AI
- Promising Topics
Interpretability Problem
Basic Types of Neurons

- Bipolar (Interneuron)
- Unipolar (Sensory Neuron)
- Multipolar (Motoneuron)
- Pyramidal Cell
A new type of neurons for machine learning

Fenglei Fan, Wenxiang Cong, Ge Wang

First published: 15 September 2017  Full publication history
DOI: 10.1002/cnm.2920  View/save citation
Cited by (CrossRef): 0 articles  Check for updates  Citation tools
XOR Gate

XOR-like function by the proposed 2\textsuperscript{nd} order neuron after 100 iterations
Lang and Witbrock reported that the standard backpropagation network cannot classify such spirals, and made a 2-5-5-5-1 network with shortcuts to solve this problem. With 2nd order neurons, we can do so with a simpler network without any shortcut.


Fan F¹, Cong W¹, Wang G¹.
Fuzzy Logic Interpretation of Artificial Neural Networks

Fenglei Fan, Student Member, IEEE, Ge Wang, Fellow, IEEE

Abstract — Over past several years, deep learning has achieved huge successes in various applications. However, such a data-driven approach is often criticized for lack of interpretability. Recently, we proposed artificial quadratic neural networks consisting of second-order neurons in potentially many layers. In each second-order neuron, a quadratic function is used in the place of the inner product in a traditional neuron, and then undergoes a nonlinear activation. With a single second-order neuron, any fuzzy logic operation, such as XOR, can be implemented. In this sense, any deep network constructed with quadratic neurons can be interpreted as a deep fuzzy logic system. Since traditional neural networks and second-order counterparts can represent each other and fuzzy logic operations are naturally implemented in second-order neural networks, it is plausible to explain how a deep neural network works with a second-order network as the system model. In this paper, we generalize and categorize fuzzy logic operations implementable with individual second-order neurons, and then perform statistical/information theoretic analyses of exemplary quadratic neural networks.

were identified [8]. However, these results do not reveal the inner working of a network, such as what and how features are extracted and propagated between layers. Gu et al. 2017 [9] offered an elegant explanation of the adversarial mechanism of GAN from the viewpoint of optimal mass transportation. Dong et al. 2017 [10] established a correspondence between deep networks and numerical ordinary differential equations to guide the structural design of a network with skip connections.

Instead of handling with existing models directly, researchers also tried to find the models that are more interpretable. For example, Wu et al. [11] utilized tree regularization to optimize a deep model with more interpretability. Fan [12] proposed a generalized hamming network based on the fact that neurons calculate generalized hamming distance when a bias is adopted. Albeit novel and interesting models developed in these pilot studies, these arts do not reveal the key mechanism based on which the existing models are so successful.
Deep Fuzzy Logic System

Logic Gates

Digital Inputs
A
B
C

Boolean Expression
\[ Q = (\overline{A \cdot B}) \cdot (\overline{A + B}) \cdot C \]

Output (Q)

Logic Diagram

Typical Truth Table

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<thead>
<tr>
<th>C</th>
<th>B</th>
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<th>Q</th>
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Fig. 3. Second-order network for recognition of Arabic digits from the MNIST dataset. The neurons in the “convolutional” layers are colored and inflated to demonstrate the types and frequencies of quadratic operations.
Software Engineering Principles

• Divide-Conquer
  Modularity, Abstraction, Cohesion & Coupling
• Formality
• Generality
• Scalability
• Reliability
• Adaptability
  Iterative Refinement, Anticipation of Change
Approximation with Width
The Expressive Power of Neural Networks: A View from the Width

Zhou Lu, Hongming Pu, Feicheng Wang, Zhiqiang Hu, Liwei Wang

(Submitted on 8 Sep 2017 (v1), last revised 1 Nov 2017 (this version, v3))

Theorem 4. Let $n$ be the input dimension. For any integer $k \geq n + 4$, there exists $F_{\mathcal{A}} : \mathbb{R}^n \to \mathbb{R}$ represented by a ReLU neural network $\mathcal{A}$ with width $d_m = 2k^2$ and depth $h = 3$, such that for any constant $b > 0$, there exists $\epsilon > 0$ and for any function $F_{\mathcal{B}} : \mathbb{R}^n \to \mathbb{R}$ represented by ReLU neural network $\mathcal{B}$ whose parameters are bounded in $[-b, b]$ with width $d_m \leq k^{3/2}$ and depth $h \leq k + 2$, the following inequality holds:

$$\int_{\mathbb{R}^n} (F_{\mathcal{A}} - F_{\mathcal{B}})^2 \, dx \geq \epsilon.$$  

(6)
Abstract

The expressive power of neural networks is important for understanding deep learning. Most existing works consider this problem from the view of the depth of a network. In this paper, we study how width affects the expressiveness of neural networks. Classical results state that depth-bounded (e.g. depth-2) networks with suitable activation functions are universal approximators. We show a universal approximation theorem for width-bounded ReLU networks: width-$(n + 4)$ ReLU networks, where $n$ is the input dimension, are universal approximators. Moreover, except for a measure zero set, all functions cannot be approximated by width-$n$ ReLU networks, which exhibits a phase transition. Several recent works demonstrate the benefits of depth by proving the depth-efficiency of neural networks. That is, there are classes of deep networks which cannot be realized by any shallow network whose size is no more than an exponential bound. Here we pose the dual question on the width-efficiency of ReLU networks: Are there wide networks that cannot be realized by narrow networks whose size is not substantially larger? We show that there exist classes of wide networks which cannot be realized by any narrow network whose depth is no more than a polynomial bound. On the other hand, we demonstrate by extensive experiments that narrow networks whose size exceed the polynomial bound by a constant factor can approximate wide and shallow network with high accuracy. Our results provide more comprehensive evidence that depth may be more effective than width for the expressiveness of ReLU networks.
Figure 1: One block to simulate the indicator function on \([a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_n, b_n]\). For \(k\) from 1 to \(n\), we "chop" two sides in the \(k\)th dimension, and for every \(k\) the "chopping" process is completed within a 4-layer sub-network as we show in Figure 1. It is stored in the \((n+3)\)th node as \(L_n\) in the last layer of \(A\). We then use a single layer to record it in the \((n+1)\)th or the \((n+2)\)th node, and reset the last two nodes to zero. Now the network is ready to simulate another \((n+1)\)-dimensional cube.

\[
L_1 = \frac{(1 - (x_1-a_1)/\delta)^+}{(1 - (x_1-b_1)/\delta)^+} - \frac{(1 - (x_1-a_1)/\delta)^+}{(1 - (x_1-b_1)/\delta)^+}
\]

\(L_1 = L_1\)
The Power of Depth for Feedforward Neural Networks

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Abstract

We show that there is a simple (approximately radial) function on \( \mathbb{R}^d \), expressible by a small 3-layer feedforward neural networks, which cannot be approximated by any 2-layer network, to more than a certain constant accuracy, unless its width is exponential in the dimension. The result holds for virtually all known activation functions, including rectified linear units, sigmoids and thresholds, and formally demonstrates that depth – even if increased by 1 – can be exponentially more valuable than width for standard feedforward neural networks. Moreover, compared to related results in the context of Boolean functions, our result requires fewer assumptions, and the proof techniques and construction are very different.

Theorem 1. Suppose the activation function \( \sigma(\cdot) \) satisfies assumption [1] with constant \( c_\sigma \), as well as assumption [2]. Then there exist universal constants \( c, C > 0 \) such that the following holds: For every dimension \( d > C \), there is a probability measure \( \mu \) on \( \mathbb{R}^d \) and a function \( g : \mathbb{R}^d \rightarrow \mathbb{R} \) with the following properties:

1. \( g \) is bounded in \([-2, +2]\), supported on \( \{ x : \| x \| \leq C \sqrt{d} \} \), and expressible by a 3-layer network of width \( C_{c_\sigma} d^{19/4} \).

2. Every function \( f \), expressed by a 2-layer network of width at most \( cc_\sigma d \), satisfies

\[
\mathbb{E}_{x \sim \mu} (f(x) - g(x))^2 \geq c.
\]
Quadratic Neural Network for Human-like Learning

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1 IBM Research Cambridge, Cambridge, MA 02142, USA
February 19, 2019

Abstract
Deep learning is the mainstream of machine learning that concerns with algorithms inspired by the functions of the human brain. Inspired by the diversity of biologic neurons, our group recently proposed quadratic neurons [1] by replacing the inner product in conventional neurons with a quadratic operation of input data, thereby enhancing the capability of the individual neuron. For instance, even a single quadratic neuron can realize the XOR logic. Along this direction, we are motivated to unlock the power of quadratic neurons in representative network architectures, towards human-like learning in the form of quadratic deep learning.

Introduction
Quadratic neuron is upgraded the conventional neuron, which integrates input data into an inner product, into the quadratic neuron that processes the n-dimension inputs as follows:

\[ h(x) = \sum_{i=1}^{n} w_i x_i + b_0 \left( \sum_{i=1}^{n} w_i x_i + b_2 \right) + \sum_{i=1}^{n} w_i x_i^2 + c \]

where only 3n parameters are used.

Algebraic Structure
We theoretically demonstrated the strength of quadratic networks [2] in the unique functional representation – a univariate polynomial of degree N can be expressed as

\[ p(x) = \sum_{i=0}^{N} a_i x^i + b_0 \left( \sum_{i=0}^{N} a_i x^i + b_2 \right) + \sum_{i=0}^{N} a_i x^i + c \]

\[ = \left( \sum_{i=1}^{N} w_i x_i + b_0 \right) \left( \sum_{i=1}^{N} w_i x_i + b_2 \right) + w_0 x^2 + c \]

Deliverable: The quadratic deep learning model will empower us to build more powerful AI tools that can help solve complex tasks.

Model Efficiency
With a huge market potential, scaling deep learning to mobile/ wearable apps has a major traction. We demonstrated merits of quadratic networks in terms of model efficiency [2].

Theorem: Given the network with only one hidden layer, there exists a function that a quadratic network can approximate it with a polynomial number of neurons but a conventional network can only do the same-level job with exponentially more neurons. Proof: The key is to utilize the properties of the Fourier transform of inner products.

Deliverable: Real-time on-site AI modules will be valuable in wearable medical devices.

Network Interpretability
Lack of the interpretability has become a primary obstacle to the wide-spread translation and development of deep learning. We propose to interpret neural networks from the perspective of engineering. We consider a deep neural network as an integrated system of fuzzy logic gates. Each quadratic module can be topologically characterized by its own spectrum [3].

Deliverable: Pushing explainable AI into IBM medical products so that trust is gained from patients and other customers.

Future Directions
- Modularize important quadratic networks
- Hybrid networks with more bio-plausibility [4]
- Develop a fuzzy theory of quadratic networks

Reference
Recently, deep learning has been playing a central role in machine learning research and applications. Since AlexNet, increasingly more advanced networks have achieved state-of-the-art performance in computer vision, speech recognition, language processing, game playing, medical imaging, and so on. In our previous studies, we proposed quadratic/second-order neurons and deep quadratic neural networks. In a quadratic neuron, the inner product of a vector of data and the corresponding weights in a conventional neuron is replaced with a quadratic function. The resultant second-order neuron enjoys an enhanced expressive capability over the conventional neuron. However, how quadratic neurons improve the expressing capability of a deep quadratic network has not been studied up to now, preferably in relation to that of a conventional neural network. In this paper, we ask three basic questions regarding the expressive capability of a quadratic network: (1) for the one-hidden-layer network structure, is there any function that a quadratic network can approximate much more efficiently than a conventional network? (2) for the same multi-layer network structure, is there any function that can be expressed by a quadratic network but cannot be expressed with conventional neurons in the same structure? (3) Does a quadratic network give a new insight into universal approximation? Our main contributions are the three theorems shedding light upon these three questions and demonstrating the merits of a quadratic network in terms of expressive efficiency, unique capability, and compact architecture respectively.
Univariate Polynomial of Order N

\[ P_N(x) = \sum_{p=1}^{N} x^p \]
Algebraic Fundamental Theorem
Particle/Factor Mathematics

\[ f(x) = f(x_1, \ldots, x_n) = \sum_{q=0}^{2n} \Phi_q \left( \sum_{p=1}^{n} \phi_{q,p}(x_p) \right) \]

Also, aided by the concept of partially separable functions, the complexity of the quadratic network can be further reduced, such as in the case of computing an \( L^{th} \) separable function. By the \( L^{th} \) separable function, we mean that \( f(x_1, \ldots, x_n) \) is \( L^{th} \) separable defined as follows:

\[ f(x_1, ..., x_n) = \sum_{l=1}^{L} \prod_{i} \phi_{li}(x_i). \]

In practice, almost all continuous functions can be represented as \( L^{th} \) separable functions, which are of low ranks at the same time.
Outline

- Deep Reason
- Experimental Results
- Explainable AI
- Promising Topics
FIGURE 2. Big picture of deep imaging – A full fusion of medical imaging and deep learning. A high likelihood is that the direct paths from data to features and actions may need an intermediate layer essentially equivalent to a reconstructed/processed image.
Rawdiomics

In Collaboration with Amber Simpson (MSK), Bruno De Man (GE GRC), Pingkun Yan (RPI), Mannudeep Kalra (MGH), et al.
End-to-end CT Imaging

Figure 4: The schematics for joined training of the reconstruction and detection neural networks. Red arrows stand for backpropagation through neural networks. $g_{\theta_1}, \ldots, g_{\theta_K}$ stand for the gradients accumulated in step 16 in algorithm 1.
Direct Sinogram Analysis

Analysis of Blood Vessel Features in the CT Sinogram via Deep Learning

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Figure 8: Example sinograms from the testing phase of the simulation study: (a) An input sinogram, (b) the label sinogram, (c) estimated sinogram and (d) error sinogram. All sinograms are displayed in a [0 - 1.0] interval.
Deep Sinogram Analysis
Energy-integrating vs Photon-counting CT

X-ray Tube → Multi-material Phantom

- Air
- Water
- Fat 50%
- Gold 10mg/ml
- Iodine 20mg/ml
- Calcium 150mg/ml

- Photon-integrating Detector

- Averaged Linear Attenuation Over Whole X-ray Spectrum

- Conventional CT

- Spectral Molecular CT

- Full Spectrum For K-edges In Many Energy Bins

- Rotation
Photon-counting Spectral CT

Acquisition of MARS Photon-counting Micro-CT Scanner

The goal is to acquire the state of the art MARS photon-counting micro-CT scanner to support major users who work on NIH-funded R01s and other research projects.

A Clinical Trial of 400 Patients in NZ (RPI plans to receive data)
Emulation on CT Benchtop (Donated by GE-GRC)

X-ray Source
60kW GE tube for 64-slice CT
High-V generator, 40-140kVp (JEDI)
Detector/DAS
Partial GE VCT-LightSpeed detector
64-slice (Z) by 128 pixels (X), 1x1 mm²

Motion Stages
Source: X, Y, Z, and φ
Detector: X, Y, and φ
Phantom: Z and φ

http://www.cirsinc.com
Simulated Example

Meng B, Yang J, Ai DN, Fu TY, Wang G
Beijing Institute of Technology, Beijing, China; Rensselaer Polytechnic Institute, Troy, NY, USA

Limerick: Lady of Niger

There was a young lady of Niger
Who smiled as she rode on a tiger;
They returned from the ride
With the lady inside,
And the smile on the face of the tiger.
Natural Language Processing (NLP)

Machine Learning for Tomographic Imaging
Ge Wang, Yi Zhang, Xiaojing Ye, & Xuanqin Mou
Semantic Tomography

Machine Learning for Tomographic Imaging
Ge Wang, Yi Zhang, Xiaojing Ye, & Xuanqin Mou
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