Mainly for Undergraduates

Math 61/65, Discrete Mathematics

A course for prospective computer science and math majors and minors, students planning to take an upper level math class, or anybody who enjoys math and would like to try something outside the standard Calculus sequence.

Discrete Mathematics provides a bridge from low level, mostly computational classes to more advanced proof-based ones. We will use the set theoretical language of mathematics and learn how to read and write proofs. We will provide an introduction to a variety of problems related to algebra, analysis, probability, geometry, topology and computer science that can be explored in depth in subsequent courses.

(Note: this course will soon be transitioned from Math 61 to Math 65.)

Math 87, Mathematical Modeling and Computation

The course is aimed at students who have completed the calculus sequence and linear algebra (Math 70) and who have an interest in computational mathematics. The course should be a good starting point for students interested in participating in the annual Mathematical Contest in Modeling.

Mathematical modeling is about the use of techniques and ideas from mathematics—and the theorems that come with them—to predict, describe, and attempt to understand the behavior of real-world systems.

The course will introduce some computational techniques as tools for the study of mathematical models. The course does not assume prior knowledge of computer programming, but in the course you will learn to write and use some code (mainly in Python, but also Matlab) to solve modeling problems.

Mathematical models can involve continuous mathematics (like PDEs and linear programming) and discrete mathematics (like graph methods and integer programming); in the course, we will study methods and models of each flavor.

Math 123, Mathematical Aspects of Data Analysis

Undergraduates who have taken multivariable calculus and linear algebra, and who are willing to do programming in MATLAB. Familiarity with any programming language will help.

Mathematics and statistics are fundamental components of data science. This is a course in mathematical data science with emphasis on theory. The course will also highlight important applications and students will have the opportunity to program some standard algorithms in MATLAB. The topics to be covered are:

(1) Principal component analysis, singular value decomposition
(2) Basic algorithms in numerical linear algebra
(3) Unsupervised learning: k-means, hierarchical clustering, density methods
(4) Spectral graph theory
(5) Nearest neighbor classification
(6) Support vector machines
(7) Neural networks and deep learning
(8) Applications to image processing, network analysis, graph matching

The goal of this course is twofold: (a) Understand important concepts in mathematical data science; (b) Learn certain applications and become competent in implementing standard algorithms.

Math 125, Numerical Analysis

This is cross-listed in Math and CS, suitable for majors/minors in those and related disciplines. Programming assignments are required (Matlab programming will be sufficient for these). Prerequisites: Calc II (required) and Linear Algebra and/or Differential Equations (required) and programming ability in C, C++, Fortran, Python, or Matlab (required).

Misha Kilmer

At this point in your mathematical career, you have developed some intuition for real numbers and their properties. You have spent some time studying real-valued functions of a real variable, including such properties as continuity and differentiability, as well as real-valued solutions to dynamical equations of importance in mathematics, science and engineering.

Real numbers are of such fundamental importance in mathematics that it may surprise you to learn that digital computers can not represent them in any exact sense. Instead, digital computers approximate real numbers by so-called floating-point numbers. The properties of floating-point numbers and functions thereof are sufficiently different from those of real numbers, that a separate branch of analysis is needed to study them, namely numerical analysis. More generally, numerical analysis is the study of algorithms for mathematical problems involving real or complex numbers, such as differentiation and integration, linear systems of equations, nonlinear algebraic equations, and differential equations.

While the approximation of real numbers by floating-point numbers is very effective for most applications, it is far from perfect. Understanding the behavior of algorithms when they are implemented in floating point arithmetic and applied to solve real world problems is crucial. Sometimes, an algorithm that is theoretically sound will perform very badly when implemented and executed on a computer. The Patriot Missile disaster and Ariane V explosion and sinking of the Sleipner A can all be traced to issues with floating-point arithmetic, and numerical errors have played a role in mishaps in parliamentary elections and stock market exchanges (see https://slate.com/technology/2019/10/round-floor-software-errors-stock-market-battlefield.html and http://www-users.math.umn.edu/~arnold/disasters/).

In spite of such worrisome issues, numerical algorithms involving floating-point numbers are widely used in science and engineering, and their importance is arguably increasing in time. They are used to predict the airflow and resultant drag of airflow over an automobile. They are used to simulate the motion of molecules in chemical reactions. They are used to turn data collected from an X-ray CT machine into a medical image for diagnostics. They are used to determine the best web pages for you to examine after you have typed search terms into Google. It has been estimated that the amount of computer cycles saved by the development and adoption of clever numerical algorithms during the twentieth century was comparable to the amount of time saved by increasing processor speeds. And very recently, numerical analysis tools have been used in the study and development of neural networks for machine learning/classification.

This course is an introduction to numerical analysis, treating linear algebra fairly lightly, instead emphasizing non-linear equations, integration, and differential equations. (For a thorough treatment of numerical linear algebra, take Math 128/CS 128.) Computer programming will be a non-trivial component of the homework.
**Math 135, Real Analysis I**

This course is aimed at math majors and minors, and is popular with quantitative economics students as well. Calc I-II required, Calc III and Linear Algebra recommended—not so much for the technical content, but rather for some experience and intuition concerning higher-dimensional Euclidean spaces, and exposure to abstract, rigorous mathematical thinking.

The basic ideas of calculus have broad-ranging extensions and generalizations that play a role in many areas of advanced mathematics (and physics). Math 135 revisits and expands on two key ideas: convergence and continuity; Math 136 addresses differentiation and integration. We will begin by revisiting the idea of a limit for sequences of numbers, and the idea of a continuous function (of a single real variable), but with careful attention to the details, and to the reasons why certain properties hold. This conceptual approach involves careful formulation and close reading of results (definitions and theorem) and rigorous justification (proofs). You will learn to think more deeply about mathematical properties, and how to communicate your reasoning clearly and precisely.

After honing these skills on relatively familiar topics, you will learn how to use your intuition, aided by rigor, to extend these ideas to new settings. For example, the existence of irrational numbers relates to the completeness of the reals; the Extreme Value Theorem relates to the compactness of sets, while the Intermediate Value Theorem relates to the idea of connectedness of sets. These ideas extend first to sets of points in the plane and higher dimensional spaces and then to the more general setting of metric spaces, which is based on the idea of an abstract “distance” between points. Ultimately, we will look at “spaces” where the “points” are actually functions, and see how the basic concepts extend from the real line to this abstract setting, and allow us to understand the functions themselves better.

You should come out of this course with two major takeaways: first, you will be able to read and write definitions, theorems and proofs with fluency and confidence, and second, you will understand how the basic (and perhaps initially baffling) ideas in Calc I-II adapt to a much broader context than just sequences, series, and functions of a (single) real variable to give us useful tools for handling many advanced mathematical topics. This generalization should also help give you a deeper understanding of the basic ideas themselves. Along the way, you may also develop an eye for aesthetic qualities—elegance, precision, and so on—in mathematical arguments.
Math 136, Real Analysis II

Prerequisite: Math 135, Real Analysis I. Math 136 prepares students for rigorous studies in geometry, topology (such as Math 285), complex variables, partial differential equations, and more advanced topics in analysis (such as Math 235).

This course is a continuation of Math 135. In Math 135 we laid the foundation of real analysis by studying the topology of a metric space and the concept of continuity. Math 136 applies these tools to three main topics: derivatives, integrals, and Fourier series, useful in fields as diverse as physics and economics.

The derivative of a function \( f : \mathbb{R}^n \to \mathbb{R}^m \) is defined to be a linear transformation, representable by a matrix of partial derivatives. Using this definition, we prove rules for differentiation (including the product rule and the chain rule) and the mean-value theorem, familiar from calculus courses. Two new results are the implicit function theorem, giving conditions under which a system of equations can be solved locally, and the inverse function theorem, giving conditions under which a function is locally invertible.

We will define the Riemann integral for functions \( f : A \to \mathbb{R} \) where \( A \) is a bounded subset of \( \mathbb{R}^n \). This will allow us to define the volume of many sets in \( \mathbb{R}^n \). For example, the volume of the unit interval \([0, 1] \subset \mathbb{R}\) is one, as you might guess. However, the set of rational numbers in \([0, 1]\) does not have volume. We define a generalization, sets of measure zero (such as \( \mathbb{Q} \cap [0, 1] \)), and use this concept to characterize the functions that are Riemann integrable. We prove some familiar theorems such as Fubini’s theorem and the change of variables formula.

Finally, we will learn about Fourier Series and Hilbert Spaces. Using concepts from real analysis, we will solve the partial differential equation describing heat distribution in an insulated rod. We anticipate proving that the solution becomes infinitely smooth as soon as the experiment starts and that the temperature of every point on the rod approaches the average temperature as time increases.

Math 145, Abstract Algebra I

This course is aimed at math, CS, and physics majors and minors. Students should have taken Linear Algebra and some proof-oriented class such as Discrete Math or Number Theory.

We will study groups and rings. Groups and rings are names for abstract structures that appear in mathematics in a variety of settings such as in the laws for addition and multiplication that you used in grade school, in the symmetries of objects, in the transformations that preserve the solutions of certain equations, in the laws of addition of points that can be defined on some curves and in many other situations. Studying groups and rings in the abstract, rather than in individual settings, provides a lot of “bang for the buck,” as the results are then applicable to all situations. We will do examples too!

As Algebra is (along with Analysis) one of the pillars of Mathematics, the class prepares you for many more advanced Math classes. Group theory that will be covered in this class is heavily used in physics and some areas of computer science.
Mostly upper-level undergraduates. Calculus II is needed, and some easy parts of Calculus III will be used (but a student can learn those as they are taking Math 165). This course is the first of a two-course sequence, Math 165/166, Probability/Statistics. It gives intuitive understanding that is helpful in understanding probability at a more advanced level.

This course will be organized not around techniques, but around twelve larger ideas and questions (roughly one per week).

Examples:
1. How good is our intuition about randomness and risk?
2. Why is there a theory of stock option pricing, but not a theory of stock pricing (assuming you don’t consider astrology a theory)?
3. Given that you test positive for a disease, how likely are you to actually have it? (Often the answer is “still very unlikely”!)
4. Why do you so often have to wait for the bus much longer than expected?
5. Why do you hear about the bell-shaped curve so often? When is its use really justified?
6. Just how unlikely are very unlikely events?
7. What exactly is entropy? How does it “measure disorder”, and how is it used in neural networks?

Thinking about these and similar guiding and motivating questions, we encounter and study all the basic ideas of probability theory, for example:
1. combinatorics in discrete probability theory,
2. discrete conditional probabilities, expectations and variances,
3. Bayes’ formula,
4. Poisson and other renewal processes,
5. the law of large numbers and the central limit theorem,
6. the Markov and Chebyshev inequalities, Chernoff bounds, large deviations,
7. probability distributions and ways of measuring how much they differ.
Undergrad and Grad

Math 185, Differential Geometry

For undergraduate and graduate students. Although it is possible to take this course having had only Calculus III (Math 42) and Linear Algebra (Math 70), it is recommended that the student has taken Real Analysis I (Math 135). This course prepares the student for a mathematical study of general relativity and also higher-dimensional differential geometry.

Loring Tu

Topology and geometry are both concerned with properties of spaces and figures in them. Broadly speaking, topology deals with “shape,” while geometry deals in addition with “size.” Differential geometry studies curves, surfaces, and their higher-dimensional analogues using techniques of differential calculus. It serves to unify various types of geometries – Euclidean, spherical, hyperbolic, and projective geometries. Developed largely in the nineteenth century, differential geometry was inspired in part by the discovery of non-Euclidean geometry and in part by problems in optics, mechanics, astronomy, and geodesy. In the twentieth century, differential geometry laid the mathematical foundation for the general theory of relativity and remains an active branch of modern mathematics.

The emphasis of this course is on concepts that are valid in all dimensions, although for pedagogical reasons we study them mainly for curves and surfaces. We will study the local and global theory of curves and surfaces, curvature, parallel translation, geodesics, and vector fields. We will be asking questions like: How does one measure the curvature of curves and surfaces? How does the curvature determine the topology? What is the analogue of a straight line on a curved surface? How does non-Euclidean geometry resolve the problem of the consistency of Euclid’s parallel postulate? The course culminates in the Gauss-Bonnet theorem, which gives a surprising link between geometry and topology.

Math 190, Topological Data Analysis

This course is aimed at juniors, seniors and graduate students who are interested in applications of topology to data. An ideal background for this course would include some linear algebra, some topology or graph theory, and some coding experience (preferably Python). If you are lacking one or more of these areas but are motivated and willing to learn new skills, feel free to reach out to the instructor.

Thomas Weighill

What is the shape of the 2016 Presidential Election? Topologists spend a lot of their time trying to understand the shape of imaginary spaces like the Klein bottle, but more recently some topologists have also become interested in studying the shape of real world data. In this course we will learn about and use some tools from the new field of Topological Data Analysis (TDA). We will do a crash course in simplicial homology so that we can understand persistent homology, the most prominent tool from TDA. We will also learn about Mapper, a data visualization tool built on topological ideas which has already been successfully applied in cancer research. We will then go hunting for datasets to apply these tools to. This will be a hands-on experience with real (sometimes messy!) data, which will allow students to build up some data science skills along the way. The goal is for students to learn some new theory, to formulate data questions and then to answer them with cutting-edge tools from TDA.
Mainly for Graduate Students

Math 220, The Top Ten Algorithms of the 20th Century

Graduate students who have taken Numerical Analysis and Numerical Linear Algebra. Advanced undergrads would be welcome if they have a solid background in Numerical Analysis and Numerical Linear Algebra.

James Adler

In the January/February 2000 issue of Computing in Science and Engineering, Jack Dongarra and Francis Sullivan chose the “top 10 algorithms with the greatest influence on the development and practice of science and engineering in the 20th century”. Since then, others have made modifications to that list. We will consider one such modification which includes algorithms such as Newton methods, Matrix Factorizations, Singular Value Decompositions, Monte Carlo methods, Fast Fourier Transforms, Krylov Subspace Methods, JPEG Compression, PageRank, the Simplex Algorithm, and the Kalman filter. We will cover each of these algorithms. Students will study the background (and possibly history) of the algorithm, and if possible implementation issues. The course will be taught in a seminar style, where students will present in class about the various algorithms.

Math 235, Graduate Real Analysis

Graduate students. Prerequisites are Math 135, Math 136, and graduate standing; or permission of instructor.

Kasso Okoudjou

This course will follow Stein and Shakarchi’s Real Analysis, Measure theory, integration, and Hilbert spaces. We will cover most of Ch 1-6, especially Ch 1-2 (Lebesgue measure and integral), Ch 3 (functions of bounded variation and differentiation theorem), Ch 4-5 (Hilbert spaces), Ch 6 (abstract measure theory and Radon-Nikodym theorem).

It is a core course to prepare you for the following topics on the real analysis qualifying exams. Measure and integration: sigma-algebras, measurable sets and functions, Lebesgue measure and integration, Monotone/Dominated Convergence, $L^p$-spaces, Fubini-Tonelli theorem, bounded variation, absolute continuity, Radon-Nikodym theorem, Carathéodory extension and abstract measure. Real functions and functionals: Banach spaces, Hilbert spaces, and topological vector spaces, linear functionals and representation theorems.
Math 240, Arithmetic Statistics

Suitable for grad students, and advanced undergraduates who have taken Math 146 and would like to see a continuation of those ideas (particularly Galois theory).

Robert Lemke Oliver

Arithmetic statistics is a relatively new area of number theory concerned with determining the “typical” properties of some class of objects. For example, If one considers a “random” polynomial with integer coefficients, how likely is it to be irreducible? What is its Galois group? Other objects of interest include number fields (finite extensions of the rationals cut out by irreducible polynomials) and Diophantine equations, particularly elliptic curves. Open questions will be highlighted along the way.

Math 255, Graduate Partial Differential Equations I

This is a graduate course, but it should be accessible to undergraduates who have taken 135 and 155.

Bruce Boghosian

Partial differential equations (PDEs) are the principal language of mathematical science, but they are also used in pure mathematics, such as in Perelman’s 2006 proof of the Poincaré Conjecture. This course will provide the student with a working knowledge of PDEs, using methods of real, complex, and functional analysis. We shall begin by analyzing the important prototypical linear PDEs for potential theory, heat transfer and wave motion, but we will move from there into discussion of quasilinear and nonlinear equations. We shall learn how to classify PDEs as elliptic, parabolic or hyperbolic, what fundamental solutions are, what well posedness means, and how to derive conditions for existence and uniqueness of their solution. This discussion will include classical results, such as the Cauchy-Kovalevskaya Theorem, but also more modern functional analytical results, such as the distinction between strong, weak, and distributional solutions, and the Lax-Milgram Theorem.

An undergraduate course in Real Analysis, similar to Math 135, is a prerequisite. There will be weekly problem sets, one midterm and a final exam. This course concentrates on the theory of linear and quasilinear PDEs; more material on nonlinear PDEs will be covered during Math 256 in the following semester. This course meets during a + block because of the considerable amount of material that must be covered.
A Lie group is a group which is also a manifold in which the group operations are $C^\infty$ maps. Lie groups are very important objects of study because they are the natural concept for describing symmetry in mathematics and physics. They are named after Sophus Lie, a Norwegian mathematician in the latter half of the nineteenth century, who studied local continuous transformation groups, particularly in the context of symmetries of differential equations.

Lie groups and the related concept of principal fiber bundle are at the center of modern differential geometry. They are also play an indispensable role in physics. For example, the irreducible representations of the rotation group SO(3) explain the distribution of atomic structures, which leads to the periodic table of elements. Gauge groups, such as $SU(3) \times SU(2) \times U(1)$ for the standard model, are important in explaining the field theories of elementary particles. Lie groups are also important in quantum field theory and string theory, the “theory of everything.”

Finally, Lie groups are beautiful because they combine geometry, algebra, and analysis in fundamental but often surprising and unexpected ways. For example, symmetric spaces are defined geometrically, their properties are studied analytically, and their classification (by Élie Cartan in the early twentieth century and more recently using Kac-Moody algebras) is completed algebraically.

In the first half of the course, we will study basic Lie theory, including the exponential mapping, Lie subgroups and subalgebras, the adjoint group, Lie transformation groups, homogeneous spaces and orbits, and compact, nilpotent and semisimple Lie groups.

In the second half of the course, we will study basic representation theory and the structure theory of semisimple Lie algebras, including Cartan’s classification.
### Tentative Course Schedule for Fall

<table>
<thead>
<tr>
<th>Course</th>
<th>Instructor</th>
<th>Days/Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math 61/65 Discrete Math</td>
<td>Montserrat Teixidor</td>
<td>D+ TR - 10:30AM-11:45AM</td>
</tr>
<tr>
<td>Math 87 Mathematical Modeling</td>
<td>George McNinch</td>
<td>R+ MW - 9:00AM-10:15AM</td>
</tr>
<tr>
<td>Math 123 Math of Data Analysis</td>
<td>Abiy Tasissa</td>
<td>M+ MW - 06:00PM-07:15PM</td>
</tr>
<tr>
<td>Math 125 Numerical Analysis</td>
<td>Misha Kilmer</td>
<td>G+ MW - 1:30PM-02:45PM</td>
</tr>
<tr>
<td>Math 135 Real Analysis I</td>
<td>Zbigniew Nitecki</td>
<td>G+ MW - 1:30PM-02:45PM</td>
</tr>
<tr>
<td>Math 135 Real Analysis I</td>
<td>Bruce Boghosian</td>
<td>D+ TR - 10:30AM-11:45AM</td>
</tr>
<tr>
<td>Math 136 Real Analysis II</td>
<td>Loring Tu</td>
<td>E+MW MW - 10:30AM-11:45AM</td>
</tr>
<tr>
<td>Math 145 Abstract Algebra I</td>
<td>Montserrat Teixidor</td>
<td>T+ TR - 9:00AM-10:15AM</td>
</tr>
<tr>
<td>Math 145 Abstract Algebra I</td>
<td>Robert Lemke Oliver</td>
<td>E+MW MW - 10:30AM-11:45AM</td>
</tr>
<tr>
<td>Math 165 Probability</td>
<td>Christoph Börgers</td>
<td>M+ MW - 06:00PM-07:15PM</td>
</tr>
<tr>
<td>Math 165 Probability</td>
<td>Alex Hening</td>
<td>A+ MW - 08:05AM-09:20AM</td>
</tr>
<tr>
<td>Math 185 Differential Geometry</td>
<td>Loring Tu</td>
<td>I+ MW - 03:00PM-04:15PM</td>
</tr>
<tr>
<td>Math 190 Topological Data Analysis</td>
<td>Thomas Weighill</td>
<td>D+ TR - 10:30AM-11:45AM</td>
</tr>
<tr>
<td>Math 220 Top Ten Algorithms</td>
<td>James Adler</td>
<td>7 W - 1:30PM-4:00PM</td>
</tr>
<tr>
<td>Math 235 Graduate Real Analysis</td>
<td>Kasso Okoudjou</td>
<td>E+MW MW - 10:30AM-11:45AM</td>
</tr>
<tr>
<td>Math 240 Arithmetic Statistics</td>
<td>Robert Lemke Oliver</td>
<td>G+ MW - 1:30PM-02:45PM</td>
</tr>
<tr>
<td>Math 255 Graduate PDE I</td>
<td>Bruce Boghosian</td>
<td>H+ TR - 1:30PM-2:45PM</td>
</tr>
<tr>
<td>Math 287 Lie Groups</td>
<td>Fulton Gonzalez</td>
<td>M+ MW - 06:00PM-07:15PM</td>
</tr>
</tbody>
</table>