Remembering Leon Ehrenpreis (1930–2010)

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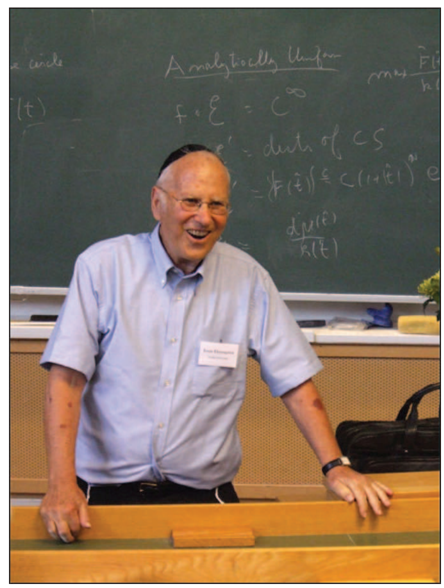

Photograph by François Rouvière.

Leon explaining his AU-spaces at the 2008 conference held in Stockholm in honor of Jan Boman's seventy-fifth birthday.

Leon Ehrenpreis: A Note on His Mathematical Work

Leon Ehrenpreis passed away on August 16, 2010, at the age of eighty, after a life enriched by his passions for mathematics, music, running, and the scriptures. I first met Leon in 1978, when I was a doctoral student under Carlos Berenstein (himself a student of Leon's), at a time when Carlos had invited him to lecture at the complex analysis seminar at the University of Maryland.

I had just begun to attempt my first (of many) readings of his work on the fundamental principle [11] and I was in awe of his presence. And yet, I discovered a gentle human being, with a passion for mathematics and a genuine desire to help the newcomer (me, in that case) to understand its mysteries.

What follows in this section is a quick overview of Ehrenpreis's work, mostly biased by my own interest and work on the fundamental principle. It is of course impossible to pay full tribute to the complexity of Ehrenpreis's work in such a short note. The subsequent sections are short remembrances, both mathematical and personal, of Leon's life, from some of his friends and collaborators.

Leon Ehrenpreis's mathematical career started with his 1953 dissertation [4], which he wrote while doing his doctoral work at Columbia University under the guidance of Chevalley. The dissertation, whose main results later appeared in [8], already shows some of Ehrenpreis's most characteristic skill in his willingness to tackle bold generalizations, as he extends distribution theory to the case in which Euclidean spaces are replaced by locally compact Hausdorff spaces denumerable at infinity and the derivations $\partial/\partial x_i$ are replaced by a countable family of local, closed operators.

He then plunged immediately into the topics that would become the main drivers for his fundamental principle. In an impressive series of papers from 1954 to 1960, Ehrenpreis addressed the *problem of division* in a variety of contexts. Specifically, he investigated in [5] the issue of surjectivity of

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Photo by Fabrizio Colombo.

Leon with his student Carlos Berenstein, Carlos's student Daniele Struppa, and Daniele's student Irene Sabadini at the 2004 conference held in Fairfax in honor of Carlos's sixtieth birthday.

partial differential operators with constant coefficients in the space $\mathcal{D}'_F$ of distributions of finite order and in the space $\mathcal{E}'$ of infinitely differentiable functions. These were the heroic years of the application of the theory of distributions to the study of the solvability of linear constant coefficients partial differential equations, and Ehrenpreis's results joined those of Hörmander [23] and Malgrange [27] to help develop the modern form of this theory. In [6] Ehrenpreis tackled the case of some special convolution equations and extended the results of L. Schwartz on exponential solutions to such equations [30], an analysis that he continued later on in [7], where he considered a larger class of spaces.

All of this culminated with his striking announcement [9] in 1960 of what became known as the fundamental principle of Ehrenpreis-Palamodov (this result was in fact also independently and concurrently discovered by Palamodov; see [28] and the Russian references therein) and whose complete proof appeared in his most important monograph [11].

The fundamental principle can be seen as a far-reaching generalization of the well-known representation theorem for solutions of ordinary, constant coefficients linear differential equations, which we teach in any introductory class on differential equations.

In order to state it (at least in a particular case), let us consider the space $\mathcal{E}$ of infinitely differentiable functions on $\mathbb{R}^n$, and let $P_1(D), \dots, P_r(D)$ be r differential operators with constant coefficients, whose symbols are the polynomials $P_1, \cdots, P_n$. Now let $\mathcal{E}^{\vec{P}}$ denote the space of infinitely differentiable functions that are solutions of the system $P_1(D)f = \cdots = P_r(D)f = 0$. The fundamental principle roughly states that it is possible to obtain a topological isomorphism between the space $\mathcal{E}^{\vec{P}}$ and the dual of a space of holomorphic functions satisfying suitable growth conditions on the union of certain varieties V_k, where $\{V_k\}$ is a finite sequence of algebraic subvarieties of $V = \{z\colon P_1(z) = \cdots = P_r(z) = 0\}$. The proof of the theorem is quite complicated, and even decades after Ehrenpreis's original announcement, it is still the subject of analysis and reinterpretations (see, for example, the chapters dedicated to the fundamental principle in [2] and in the third edition of Hörmander's monograph on several complex variables [25]). The crux of the proof, however, is the understanding of the nature of the growth conditions that must be satisfied by holomorphic functions on the variety V for the duality to take place. Such growth conditions must be imposed not just on the functions, but also on suitable derivatives that somehow incorporate the multiplicities related to the polynomials P_j.

Maybe the most important consequence of this result is the representation theorem, which states that every function f in $\mathcal{E}^{\vec{P}}$ can be represented as

$$f(x) = \sum_{k=1}^{t} \int_{V_k} Q_k(x) \exp(iz, x)\, d\mu_k(z),$$

where the Q_ks are polynomials and μ_ks are bounded measures supported by V_k.

It is important to point out that the fundamental principle actually extends to rectangular systems of differential equations and that it holds for a very large class of spaces (what Ehrenpreis called *analytically uniform spaces*), a category that includes Schwartz's distributions, Beurling spaces, holomorphic functions, etc. (when the objects in these spaces are generalized functions, the representation theorem stated above has to be suitably interpreted, of course).

One of the common themes in most of Ehrenpreis's work in those years was a sort of philosophical principle that held that one could extend most structural results that hold for holomorphic functions to spaces of solutions of systems of linear constant coefficient differential equations satisfying suitable conditions.

Maybe the most striking such result was his beautiful and elegant proof of the classical Hartogs' theorem on the removability of compact singularities for holomorphic functions. By essentially using the arguments he had developed in the course of his study of division problems, Ehrenpreis demonstrated [10] that Hartogs' phenomenon is not a peculiarity of holomorphic functions in several variables but in fact holds for more general spaces of functions that are solutions of systems of differential equations (or even convolution equations), satisfying some specific algebraic properties. The paper is beautiful and elegant and shows Ehrenpreis's talent for simplicity and anticipates the kind of algebraic treatment of systems of differential equations that became so important in years to come.

The interest of Ehrenpreis in removability of singularity phenomena is also apparent in another beautiful series of papers ([12], [13], [14], [15]) dealing with elegant and surprising variations on the edge-of-the-wedge theorem.

I am obviously unable to touch upon all the important work of Ehrenpreis, but it would be impossible not to mention his interest in the Radon transform, which absorbed much of his mathematical efforts in the last ten or so years. His work on this topic culminated in his second monograph [16], in which Ehrenpreis introduces a generalized definition of the Radon transform in terms of very general geometric objects defined on a manifold (what he calls spreads). Just like his previous monograph [11], this more recent work brings a new and invigorating perspective to his subject matter. And as in the case of [11], this work will provide mathematicians with ideas and challenges that should keep them busy for a long time to come. As Ehrenpreis's former student Carlos Berenstein states in his review [1] of [16] "[this is] a book that is worth studying, although mining may be a more appropriate word, as the reader may find the clues to the keys he's searching for to open up subjects that are seemingly unrelated to this book. Thus, one finds at the end that the title is justified."

Hershel Farkas

A Remembrance

Leon Ehrenpreis was one of the leading mathematical figures of the past century. In many areas of mathematics, specifically differential equations, Fourier analysis, number theory, and geometric analysis, his name is a household word. He was a person who could discuss mathematical problems from a very wide variety of subfields and directly and indirectly had a great influence and impact on mathematicians and mathematics. He had the ability of getting to the core of a problem and usually the ability to give suggestions for possible solutions; sometimes he left these for others, worked jointly with other people toward the solution, or just solved the problem himself. Ehrenpreis's diversity extended way beyond mathematics. He was a pianist, a marathon runner, a Talmudic scholar, and above all a fine and gentle soul. My personal association with Leon Ehrenpreis goes back over fifty years. We began as teacher-student when I was still doing my undergraduate degree. I heard about a famous, brilliant young mathematician who was going to give a course entitled *Mathematics and the Talmud* and I decided to register for it. After the semester I asked him whether he would be willing to write a letter supporting my application to graduate school, and he told me that I would be better off asking a person more familiar with my mathematical abilities. After that I did not have much personal contact with Leon until after I finished my graduate work and our paths crossed again. I was working on problems related to compact Riemann surfaces, and we would meet now and then. Leon was also very much interested in this subject (Ehrenpreis conjecture, Schottky problem) and always expressed an interest in what I was doing. We even wrote two papers together ([20], [17]), but this is not the measure of the influence he had on me. In fact, my book with Irwin Kra, *Theta Constants, Riemann Surfaces and the Modular Group*, [21] grew out of a conversation with Leon. A visit to New York would never be complete without meeting Leon. In fact I arrived in New York on August 15 planning to meet with Leon in the middle of the week to discuss my current work, but he passed away on Monday, August 16. Mathematics has lost one of its finest practitioners. He will be sorely missed by the mathematical community as both a scholar and a gentleman.

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Takahiro Kawai

The Fundamental Principle, Hamburger's Theorem and Date Line

When I was a graduate student (late sixties), the "fundamental principle of Ehrenpreis" [9] was really a guiding principle for many young analysts, including me. It aimed at, and succeeded in, analyzing general systems of linear differential equations with constant coefficients, in the days when a single equation or a determined system was the main target of specialists in differential equations, and the cohomological machinery was not in the toolboxes of most of them. Thus the publication of his book [11] had been yearned for; actually I asked a bookseller to get it by airmail as soon as it appeared—it was an exceptional case as one U.S. dollar equaled 360 yen in 1970. Unfortunately I could not indulge in reading [11] as I had hoped; in 1970 and 1971 I was totally absorbed in the collaboration with Sato and Kashiwara to complete [29], which was a successor to [9] in the sense that it presented the microlocal structure theorem

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Photograph by Yael Ehrenpreis Meyer.

Leon with Jan Boman at the Stockholm conference.

for a general system of linear differential equations, not necessarily with constant coefficients, at a generic point of its characteristic variety. Still, I have continued to enjoy browsing through [11] occasionally; the book is full of the ideas and dreams of Leon Ehrenpreis. When I browse in the pages of [11], I feel as if I were chatting with Leon. The paper [26] is an example of outcomes of such chatting.

When I first met Leon in 1973 in Kyoto, I got the impression that he was a kind man of sincerity. The impression has continued until now. In ending this memory I note two incidents that endorse the impression. When I explained the prototype of $\mathbb{R}$-holonomic complexes to Leon in 1980, he immediately noticed its relevance to Hamburger's theorem and he kindly expounded the theorem and related works of Hecke and Weil. I cannot forget the warm atmosphere full of intellectual curiosity, which led to our paper [19]. Another incident I note is that I once happened to notice that he had not taken anything for two days and that the reason was that he was dubious about the date of the fast day in Kyoto due to the effect of the International Date Line.

Thus I have learned much from Leon both in mathematics and in daily life. Many, many thanks, Leon!

Peter Kuchment

Leon Ehrenpreis—A Friend and Colleague.[1]

It is hard for me to believe that one now has to use the past tense when talking about Leon Ehrenpreis. The thought of him being gone has not yet sunk in.

I have been familiar with some of Leon's outstanding results (e.g., existence of fundamental solutions) for a very long time, at least since the early 1970s. His work on what he called the fundamental principle, as well as related papers and books by B. Malgrange and V. Palamodov, played a crucial role in my work with L. Zelenko in the late 1970s on periodic PDEs. We thus had to learn Leon's techniques in detail. It was not an easy task, in part since Leon always preferred unusual terms and notations, but once one got through these minor hurdles, one was always rewarded by an abundance of wonderful ideas.

I met Leon for the first time in 1989 at the integral geometry and tomography conference in Arcata, California. It was my very first trip outside the former USSR, and it felt like being in a dream. First of all, the constructive proof that the world outside the iron curtain truly did exist was a revelation! Another shock during my first visit and my emigration soon afterward was that names like Leon Ehrenpreis, Peter Lax, Louis Nirenberg, and other luminaries, which obviously existed only on book covers, or at least referred to semi–gods somewhere well above this Earth, corresponded to mere mortals, who not only existed in the flesh but were not that old at all and still very active. Meeting Leon in Arcata was my first experience of this kind. (I also met Sigurdur Helgason there, but his existence had been proven to me earlier indirectly, when I communicated with him while translating his book *Groups and Geometric Analysis* [22] into Russian.) Another happy discovery was that, unknown to me at that time, Leon had worked on integral geometry, and I had just started working in computed tomography, in which integral geometry is one of the main mathematical tools. This is how our collaboration started.

Just like everyone else, I loved Leon from the first encounter. His unfailing cheerful disposition and his abundant eagerness to discuss any kind of mathematics at any time made every occasion we met feel like a holiday. And we met quite a few times over the past twenty years, Leon visiting me in Kansas and Texas on a regular basis, seeing each other at conferences, and communicating by phone and email. I have also had the privilege of writing joint papers together with him and contributing (jointly with Todd Quinto) a chapter to his book [16] on the Radon transform.

Everyone who knew Leon Ehrenpreis realized that his brain was engaged all the time in some variant of intellectual activity—be it mathematics, or the Talmud, or music. It is well known that he was an avid runner and had run the New York marathon every year since its inception in 1970 till 2007. He also liked to run during his visits, so when he visited me in Wichita, Kansas, I would sometimes pick a room for him in a hotel seven

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[1] *I am grateful to P. Deift, M. Mogilevsky, and D. C. Struppa for useful comments and corrections.*

miles away from the campus, with a sufficiently attractive route to run between the two. So, after his lecture, or just a working day, he would give me his things to take back to the hotel, while he would run. Every time I would meet him after the run, he would have some new ideas (and he had so many great ideas!) about the problem we were working on at the time. Once, when he came back and I was waiting for him in the hotel's lobby, the receptionist at the front desk asked him: "Did you really run all the way from the campus?" Leon's reply was: "What else could I do? He refused to give me a ride"—and he pointed at me. I think I lost all the receptionist's respect at that time. This was not a one-time event; Leon always liked to crack or to hear a good joke. He was smiling most of the time that I saw him. It was a joy to discuss with him not only mathematics, but also religion, music, or anything else. What made this even more enjoyable was that in my experience he never imposed his opinions, beliefs, or personal problems (and he unfortunately had quite a few) on others. It was relaxing to talk to him. He must have been a wonderful rebbe. Over the years, he became our dear family friend.

Leon always liked a good story and had some of his own to tell. My favorite, which I tell to students often, was about him teaching a calculus class many years ago. As any good teacher would do, he tried to lead his students, whenever possible, to the discovery of new things. So, he once said: "Let us think, how could we try to define the slope of a curve?" "What is there to think about?" was the reply from one smart student, "it says on page 52 of our textbook that this is the derivative." "Well," replied Leon, "I haven't read till page 52 yet." The result was that the class complained to the administration that they were given an unqualified teacher. So much for inspiring teaching; it can backfire!

I miss Leon, a great mathematician and human being, and hope that he dwells happily in the house of the Lord forever.

Photograph by Yael Ehrenpreis Meyer.

Leon at the Stockholm conference in front of the Royal Palace.

Eric Todd Quinto

A Remembrance

Before I met Leon Ehrenpreis I was a little apprehensive. He had theorems named after him, and I knew he was a brilliant mathematician. His work on existence of solutions to PDEs helped revolutionize the field, and I wondered what its creator would be like. Leon immediately put me at ease with his warm enthusiasm and love for mathematics. When we talked about mathematics he described the big picture, and he illustrated his points with examples that brought the ideas down to earth. Leon's enthusiasm was contagious, and it got even skeptical listeners involved and intrigued. I especially appreciated Leon's respect and concern for the person he was talking with; he wanted the listener to be as engaged and interested as he was. His daughter Yael told me that he was exactly the same way when he helped her with math homework.

Leon's mathematics reflected his emphasis on unifying principles. In Leon's book [16] *The Universality of the Radon Transform*, he developed several overarching ideas and used them to understand properties of the transforms, such as range theorems and inversion methods. One such overarching idea he defined is the nonparametric Radon transform using spreads (a collection of foliations of Euclidean space, the leaves of which depend on a *spread* parameter). For the hyperplane transform, the spread is defined by a specific hyperplane L_0 through the origin (or element of the Grassmannian $G(n, n-1)$, and the spread variable is a point s in $L_0^\perp$: the spread is defined for each L_0 by $s \mapsto s + L_0$. After ten pages of the book, he is considering other manifolds and spread variables, and he is relating the geometric construction to differential equations and groups. The book draws connections between several fields, including complex variables, PDE, harmonic analysis, number theory, and distribution theory—all of which have benefited from his contributions over the years. I remember one talk he gave at Harvard while he was writing the book. He pointed to about three hundred pages of notes on the lectern and told us with a broad smile and hearty laugh that he was going to talk about all that. Worried that we would be there for a while, we laughed a little nervously. However, if I am remembering right, he covered a lot of material

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but ended more or less on time. Peter Kuchment and I wrote an appendix to the book, and this, too, was a pleasure.

Leon was a rabbi, and some of our most lively conversations occurred when I asked him simple questions about Jewish law. Leon would launch into an energetic description of what the rabbis thought about a point and how they resolved seeming inconsistencies. He always made sure the conversation was a dialogue and that I was engaged. He cared deeply about Jewish law, and he wanted to share it with me.

I will miss Leon's theorems and mathematical taste, as well as our conversations. His ideas are important, but so was his way of communicating those points and sharing his enjoyment of them. I will miss Leon's enthusiasm for math and life, his smile, and his *menschlichkeit*.

Shlomo Sternberg

A Personal Recollection

I will leave it to others to describe the extremely distinguished mathematical career of Leon Ehrenpreis. In fact, Leon, Victor Guillemin, and I wrote one joint paper together [18] of which I am still proud, but this reflects only a very tiny fraction of his great mathematical achievements. I will concentrate on the personal.

I first met Leon Ehrenpreis in the early 1950s at Johns Hopkins University, where I was a student and he an instructor. We became close personal friends, and then, when I married Aviva, he became a close friend of ours, a friendship that lasted from the time that we were teenagers until his death. Thinking back through the years, I can't recall a single time, no matter how trying the circumstances may have been, whether casual or serious, that his voice, his eyes, his whole demeanor conveyed less than deep warmth, profound generosity, an optimism, a hopefulness that was pure Leon.

When we were young, "pure Leon" might include a dash of madcap charm, a directness, a boyish whimsy, a ruefulness, that belied his distinguished mathematical achievements. His style was not professorial. He was not into style or image—then or ever. Leon retained and presented an honesty, a disarming forthrightness, a genuineness, a profound generosity and sheer vitality that he carried with him all of his life. He was certainly extremely generous to us at crucial junctures in our lives.

And it was, and is, this vitality that perhaps for us best describes Leon. The years passed; life transpired with its joys and sorrows. For Leon and his family, the sorrows were of such immensity that would otherwise crush anyone. But Leon bore his with unimaginable courage and responsibility. Courage that, we dare say, none of us could have possibly comprehended, let alone mustered. But despite it all, and no matter what transpired, Leon retained every bit of the vitality of our earlier years. His mathematical work continued. His, along with Ahava's, loving care and unstinting dedication to his family continued. His kindness and loyalty to us, his friends, continued. It was who he was.

To think now that this vitality is gone from us is very hard to accept. We'll always cherish our recollections of him—even as an attempt, however illusive, to conjure up a whiff of his vitality. Leon was one of a kind. We know he is irreplaceable. And we miss him.

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Alan Taylor

Remembrances of Leon Ehrenpreis

I first met Leon in the summer of 1966 at an AMS summer conference, "Entire Functions and Related Parts of Analysis", in La Jolla, California. I had just completed my thesis, which was about problems in spaces of entire functions that fit his theory of *analytically uniform spaces*, and I had studied some of his important papers on division problems and their applications to constant coefficient partial differential operators. So it was very exciting to meet and talk with him. It was also where I had my first encounter with his kindness, as he suggested that I apply to spend a postdoctoral year at the Courant Institute, where he was a professor. It was during that year, 1968-1969, that I really got to spend a lot of time talking with him and met his student at that time, Carlos Berenstein. Thinking back on those times is bittersweet; it is sad now because of Leon's death and doubly so because Carlos is too ill to share memories of that special year. But it also reminds me of this most interesting and fun year of my professional life. Carlos was finishing his doctoral thesis at Courant and helping Leon with the final editing of his book on Fourier analysis [11]. Leon had just moved to Yeshiva University in uptown New York City, where he was giving a course on the book. Both Carlos and his wife and I and my family lived in New York University-owned housing in Greenwich Village, so each Thursday Carlos and I would take the A train uptown to spend the day with Leon, attending his class and talking about mathematics.

I really saw Leon's style of doing mathematics in that class. He was always interested in the

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Leon discussing mathematics with Dennis Sullivan and John Morgan at Columbia University, 2006.

fundamental reasons that theorems were true and in illustrative examples but less interested in the details. It seemed to me that he could look at almost any problem in analysis from the point of view of Fourier analysis. Indeed, his book on Fourier analysis, in addition to presenting the proof of his most important contribution, the fundamental principle, contains chapters on general boundary value problems, lacunary series, and quasianalytic functions in which this general point of view is explored. This style of inquiry is present in much of his work, for example, that on the "Edge of the Wedge theorem" and its extension to some systems of constant coefficient partial differential equations. Leon was doing mathematics 100% of the time I spent around him and I think it was true always, especially when riding the train and in his jogging. After that year, I saw Leon about every year or two, mostly when he would visit our department in Ann Arbor. I was lucky that his wife, Ahava (nee) Sperka, was from a family that lived in a Detroit suburb, so that family visits were always a good occasion for Leon to visit Ann Arbor and our department.

I want to pass along two things I learned about Leon over the years that I think are worth remembering. First, I learned from my advisor, Lee Rubel, that Leon came from a remarkable undergraduate class at City College of New York, a fact that Harold Shapiro also mentions in his article [31] remembering Allen Shields, my colleague for many years at Michigan. In that class, a year or two around the year 1949, there was a remarkable group of young mathematicians who interacted with one another. The group included Leon Ehrenpreis, Donald Newman, Lee Rubel, Jacob Schwartz, Allen Shields, Harold S. Shapiro, Leo Flatto, Martin Davis, and Robert Aumann. What an outstanding group of mathematicians! Also in the group was David Finkelstein (physics). Several times I heard stories from Rubel and Shields about the stimulating atmosphere and competition between these very talented youths.

A second vivid memory I have is of a conversation with Leon sometime in the 1970s, in large part because it was so influential on my own work. Throughout the 1970s, I often talked with Leon about the problem of determining which constant coefficient partial differential operators had a continuous linear right inverse on the space of infinitely differentiable functions. At some point I asked him how, being a student of Claude Chevalley, he had come to work on problems that led to the fundamental principle. He told me that Chevalley had suggested he write to Laurent Schwartz asking for thesis-problem suggestions. Schwartz had written back with a list of several questions about partial differential operators along with what he knew about them at the time. These included the fundamental questions, answered by Leon, Malgrange, and Hörmander in the 1950s, that now underlie the modern theory of linear constant coefficient partial differential operators. He also told me that the list from Schwartz included the right inverse question that we had been discussing, as well as the question of surjectivity of such operators on the space of real analytic functions. (The surjectivity on real analytic functions was solved in the negative by De Giorgi and Cattabriga [3] with a characterization of the surjective operators being given by Hörmander in 1973.) The right inverse question was solved by Meise, Vogt, and myself around 1988, and questions that arose from this work have occupied me for the last twenty years. So it is indeed the case that Leon Ehrenpreis had a profound influence on my entire professional life.

Leon was a kind man, always interested in talking about mathematics. I never saw him without a smile on his face, and I never heard an unkind word from him. He was generous with his mathematical suggestions and a great friend. I cannot say that I was personally close to him in that I knew much about his private life, but I still remember many little things. Like the fact that he always drank tea when we worked, that he wanted to have enough children to form a baseball team, and that he always called me "Alan", my boyhood name. I will greatly miss him.

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