Math 250 — Introduction to Geometric Group Theory — Fall 2010

Course Information

BLOCK: D+: TTh 10:30 - 11:45 am
INSTRUCTOR: Kim Ruane
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OFFICE HOURS: TBA pm
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PREREQUISITES: Graduate level algebraic Topology or consent.

SUGGESTED TEXTS: *Metric Spaces of Non-positive Curvature* by M. Bridson and A. Haefliger, Springer-Verlag (1999); *Groups, Graphs, and Trees* by J. Meier, London Math. Soc. Texts, no. 73 (2008). *Trees* by J.P. Serre, Springer-Verlag (1980). Other notes and such will be posted on my webpage: *http://www.tufts.edu/ kruane01/*

COURSE DESCRIPTION:

The main idea of Geometric Group Theory is to study an infinite, finitely generated group G by viewing G as a metric space. If S is a finite generating set for G, one can define a locally finite metric graph $\Gamma(G,S)$ called the Cayley graph of G with respect to the generating set S. This graph is a proper geodesic metric space on which G acts by isometries. One hopes to recover algebraic properties of G by studying geometric properties of $\Gamma(G,S)$. Sometimes a richer geometry gives more information about G so one can study any space X on which G acts *geometrically*. A geometric action of a group G on a metric space X is a properly discontinuous, cocompact action by isometries. This is very much like a covering space action but not necessarily free. The prototypical example of such an action is $G = \pi_1(M)$ where M is a compact Riemannian manifold of negative or nonpositive curvature acting on $X = \tilde{M}$, the universal cover of M.

This area of mathematics is relatively new - it really began in the early 80's with the seminal paper of Gromovs, *"Hyperbolic Groups"*. There are many possible roads one could take in an introductory course on the topic. We will begin with the study of Cayley graphs, quasi-isometry, and geometric group actions. I plan to develop the basic theory of word hyperbolic groups and prove that several quite different looking definitions of word hyperbolicity are in fact equivalent. From there, who knows where we will go!

COURSE REQUIREMENTS: There will be written homework assignments given throughout the term. I encourage you to work together on these assignments, however I would like the final write-up to be done on your own. You will be required to choose a research paper (or part of one) to present in class. I will also ask students to "volunteer" to take notes and TeX them.

LEARNING OBJECTIVES: This course is aimed mainly at students who are ready to do research. Through the written assignments, the students will be judged on all of the points listed in the learning objectives parts 1 and 2. The final project of presenting a current research paper will provide an opportunity for me to judge the objectives in parts 3 and 4. The numbers referred to are in the document at: *http://math.tufts.edu/?pid=188*