Math 250 - Homework 2

(1) Prove that the composition of two quasi-isometries is a quasi-isometry. Suppose X, Y, Z are metric spaces, $\phi : X \to Y$ is a quasi-isometry with constant λ and $\psi : Y \to Z$ is a quasi-isometry with constant λ'. Show $\psi \circ \phi : X \to Z$ is a quasi-isometry with constant Λ where you find Λ in terms of λ and λ'.

(2) Prove that commensurability is an equivalence relation for groups.

(3) Prove that $\mathbb{E}^1$ is not quasi-isometric to $[0, \infty)$ (with the induced metric on this coming from $\mathbb{E}^1$).

(4) Suppose $\phi : \Gamma_1 \to \Gamma_2$ is a homomorphism between finitely generated groups. Show that if ϕ is a quasi-isometric embedding then $\ker(\phi)$ is finite and that ϕ is a quasi-isometry if and only if $\ker(\phi)$ and $\Gamma_2/\mathrm{i}m(\phi)$ are both finite.

(5) For integers $m, n \geq 3$ show that T_m is quasi-isometric to T_n. Note: you could do this directly, but I would rather you use SOME group theory....