Course Information Book
Department of Mathematics
Tufts University
Fall 2018

This booklet contains the complete schedule of courses offered by the Math Department in the Fall 2018 semester, as well as descriptions of our upper-level courses and a some lower-level courses. For descriptions of other lower-level courses, see the University catalog.

If you have any questions about the courses, please feel free to contact one of the instructors.

Descriptions may not be available for all courses*

FALL 2018 SCHEDULE					
COURSE	TITLE	INSTRUCTOR	BLOCK	TIMES	Max Enr
MATH 0010-01	An Introduction to Wealth Inequality	Bruce Boghosian	R+	MW 9:00-10:15	40
MATH 0010-02	Quantitative Reasoning	Mary Glaser	J+	TTh 3:00-4:15	40
MATH 0019-01	Mathematics of Social Choice	Linda Garant	D+	TTh 10:30-11:45	43
MATH 0019-02	Mathematics of Social Choice	Zac Faubion	T+	TTh 9:00-10:15	43
MATH 0021-01	Introductory Statistics	Linda Garant	H+	TTh 1:30-2:45	35
MATH 0032-01	Calculus I	Michael Chou	E	MWF 10:30-11:20	90
MATH 0032-1RA	- Calculus I Recitation Chou	TA	BR	Th 8:30-9:20	32
MATH 0032-1RB	- Calculus I Recitation Chou	TA	FR	Th 12:00-12:50	32
MATH 0032-1RC	- Calculus I Recitation Chou	TA	JR	Th 3:00-3:50	32
MATH 0032-02	Calculus I	Ky Tran	I	MW 3:00-3:50pm, F 3:30-4:20pm	90
MATH 0032-2RA	- Calculus I Recitation Kaufmann	TA	AW	8:30-9:20	32
MATH 0032-2RB	- Calculus I Recitation Kaufmann	TA	EW	W 10:30-11:20	32
MATH 0032-2RC	- Calculus I Recitation Kaufmann	TA	KW	W 4:30-5:20	32
MATH 0032-03	Calculus I	Gail Kaufmann	B	TThF 8:30-9:20	90
MATH 0032-3RA	- Calculus I Recitation Ruane	TA	CT	T 9:30-10:20	32
MATH 0032-3RB	- Calculus I Recitation Ruane	TA	FT	T 12:00-12:50	32
MATH 0032-3RC	- Calculus I Recitation Ruane	TA	JT	T 3:00-3:50	32
MATH 0032-04	Calculus I	Kim Ruane	A	MW 8:30-9:20 Th 9:30-10:20	90
MATH 0032-4RA	- Calculus I Recitation Tran	TA	GW	W 1:30-2:20	32
MATH 0032-4RB	- Calculus I Recitation Tran	TA	KW	W 4:30-5:20	32
MATH 0032-4RC	- Calculus I Recitation Tran	TA	MW	W 6:00-6:50	32
MATH 0034-01	Calculus II	Zac Faubion	H	TTh 1:30-2:20 F 2:30-3:20	90
MATH 0034-1RA	- Calculus II Recitation Faubion	TA	AW	W 8:30-9:20	30
MATH 0034-1RB	- Calculus II Recitation Faubion	TA	GW	W 1:30-2:20	30
MATH 0034-1RC	- Calculus II Recitation Faubion	TA	KW	W 4:30-5:20	30
MATH 0034-02	Calculus II	Gail Kaufmann	C	TWF 9:30-10:20	90
MATH 0034-2RA	- Calculus II Recitation Kaufmann	TA	FT	T 12:00-12:50	30
MATH 0034-2RB	- Calculus II Recitation Kaufmann	TA	JT	T 3:00-3:50	30
MATH 0034-2RC	- Calculus II Recitation Kaufmann	TA	LT	T 4:30-5:20	30
MATH 0034-03	Calculus II	Xiaozhe Hu	B	TThF 8:30-9:20	90
MATH 0034-3RA	- Calculus II Recitation Hu	TA	CT	T 9:30-10:20	30
MATH 0034-3RB	- Calculus II Recitation Hu	TA	FT	T 12:00-12:50	30
MATH 0034-3RC	- Calculus II Recitation Hu	TA	HT	T 1:30-2:20	30
MATH 0042-01	Calculus III	Montserrat Teixidor	B	TThF 8:30-9:20	90
MATH 0042-1RA	- Calculus I Recitation Teixidor	TA	CT	T 9:30-10:20	30
MATH 0042-1RB	- Calculus I Recitation Teixidor	TA	FT	T 12:00-12:50	30
MATH 0042-1RC	- Calculus I Recitation Teixidor	TA	JT	T 3:00-3:50	30
MATH 0042-02	Calculus III	Caleb Magruder	C	TWF 9:30-10:20	90
MATH 0042-2RA	- Calculus I Recitation Magruder	TA	GW	W 1:30-2:20	30
MATH 0042-2RB	- Calculus I Recitation Magruder	TA	KW	W 4:30-5:20	30
MATH 0042-2RC	- Calculus I Recitation Magruder	TA	MW	W 6:00-6:50	30
MATH 0044-01	Honors Calculus III	Zbigniew Nitecki	T+	TTh 9:00-10:15	40
MATH 0050-01	Combinatorics	Loring Tu	I+	MW 3:00-4:15	40
MATH 0051-01	Differential Equations	George McNinch	C	TWF 9:30-10:20	80
MATH 0051-1RA	- Diffi Q Recitation McNinch	TA	GW	W 1:30-2:20	30
MATH 0051-1RB	- Diffi Q Recitation McNinch	TA	KW	W 4:30-5:20	30
MATH 0051-1RC	- Diffi Q Recitation McNinch	TA	MW	W 6:00-6:50	30
MATH 0061-01	Discrete Mathematics	Montserrat Teixidor	F+TR	TTh 12:00-1:15	34
MATH 0070-01	Linear Algebra	Mary Glaser	F	TThF 12:00-12:50	43

MATH 0070-02	Linear Algebra	Eunice Kim	B+TR	TTh 8:05-9:20	43
MATH 0070-03	Linear Algebra	Todd Quinto	J+	TTh 3:00-4:15	43
MATH 0070-04	Linear Algebra	Genevieve Walsh	G+	MW 1:30-2:45	43
MATH 0087-01	Math Modeling & Computation	Caleb Magruder	D+	TTh 10:30-11:45	34
MATH 0121-01	Mathematical Neuroscience	Christoph Borgers	C	TWF 9:30-10:20	34
MATH 0123-01	Math Aspects of Data Analysis	James Murphy	N+	TTh 6:00-7:15	34
MATH 0126-01	Numerical Analysis	Bruce Boghosian	G+	MW 1:30-2:45	34
MATH 0135-01	Real Analysis I	Alexandru Hening	H+	TTh 1:30-2:45	40
MATH 0135-02	Real Analysis I	Alexandru Hening	J+	TTh 3:00-4:15	40
MATH 0145-01	Abstract Algebra I	Robert Kropholler	I+	MW 3:00-4:15	40
MATH 0145-02	Abstract Algebra I	Robert Kropholler	K+	MW 4:30-5:45	40
MATH 0150-01	Mathematical Cryptography	Michael Chou	G	MWF 1:30-2:20	34
MATH 0150-02	Numerical Optimization	Xiaozhe Hu	F+TR	TTh 12:00-1:15	34
MATH 0155-01	Nonlinear Dynamics & Chaos	Eunice Kim	L+	TTh 4:30-5:45	34
MATH 0161-01	Probability	Alexandru Hening	D+	TTh 10:30-11:45	40
MATH 0161-02	Probability	Ky Tran	E+	MW 10:30-11:45	40
MATH 0167-01	Differential Geometry	Loring Tu	M+	MW 6:00-7:15	34
MATH 0211-01	Analysis	Fulton Gonzalez	D+	TTh 10:30-11:45	34
MATH 0215-01	Algebra	David Smyth	I+	MW 3:00-4:15	34
MATH 0219-01	Algebraic Topology	Genevieve Walsh	R+	MW 9:00-10:15	34
MATH 0250-01	Harmonic Analysis	Fulton Gonzalez	N+	TTh 6:00-7:15	34
MATH 0251-01	Partial Differential Equations I	James Adler	H+	TTh 1:30-2:45	34
MATH 0295-01	Thesis	Advisor	ARR	-	50
MATH 0297-01	Graduate Research	Advisor	ARR	-	50
MATH 0340-01	Tufts Abroad Program	-	ARR	-	50
MATH 0401-PT	PT MS Degree Continuation	Advisor	ARR	-	50
MATH 0402-FT	FT MS Degree Continuation	Advisor	ARR	-	50
MATH 0501-PT	PT PhD Degree Continuation	Advisor	ARR	-	50
MATH 0502-FT	FT PhD Degree Continuation	Advisor	ARR	-	50

Math 10 An Introduction to Wealth Inequality Fall 2018

Course Information

Block: R+ (MW 9:00-10:15)
Instructor: Bruce Boghosian
Email: `bruce.boghosian@tufts.edu`
Office: Bromfield-Pearson 211
Office hours: TBA
Prerequisites: High school mathematics
Text: TBA, on-line material, plus instructor's notes

Course description:

In 2010, there were 388 billionaires in the world whose combined wealth exceeded that of half the earth's population. Today, that number is under 50, and all indications are that it continues to decrease. The enormous concentration of wealth and the unchecked growth of inequality have emerged as crucial social issues of our time. To what extent can mathematics help shed light on this problem?

In this interdisciplinary course, which requires only high school mathematics as a prerequisite, we will learn to think about wealth distribution in a quantitative fashion. We will learn the difference between wealth, money and income, and we will learn how these things are measured by banks, governments and international institutions. We will survey historical thought on this subject from mathematical, economic and philosophical perspectives.

Some of the quantitative ideas in this course will be introduced by computer simulation of idealized mathematical models. No prior knowledge of a computer language is required, but instruction will be provided for the use of *Mathematica*©, which is available to all Tufts students. We will ask questions such as,

- Can inequality be quantified? What properties should a mathematical measure of inequality have to capture our intuitive notion of the concept?
- Can idealized mathematical models, such as agent-based models, describe the current distribution of wealth with any accuracy?
- Are market economies naturally stable, or is continuous government intervention needed to keep them stable?
- What ethical tools exist to determine the morality of decisions that societies make about wealth distribution and wealth inequality?
- Should societies attempt to manage their levels of inequality? If so, what public policy tools do they have at their disposal for doing so? If not, what, if anything, should be done about runaway concentration of wealth?

What we learn along the way will raise deep mathematical, economic, and ethical questions about the way that human society has chosen to allocate limited resources amongst people and populations. Our emphasis will be on how mathematical thinking contributes to this critically important conversation.

Some description will be given of available databases for the study of wealth distribution, including that maintained by the Federal Reserve and the U.S. Census Bureau, as well as international data available, for example from the *World's Top Incomes Database*.

Only high school mathematics, and no prior background in economics is assumed. There will be weekly problem sets, at least one midterm and a final exam.

Math 50 — Combinatorics — Fall 2018

Course Information

BLOCK: M+ (Mon, Wed, 3:00–4:15 p.m.)
INSTRUCTOR: Loring Tu
EMAIL: `loring.tu@tufts.edu`
OFFICE: Bromfield-Pearson 206
OFFICE HOURS: (Spring 2018) Mon 2:50–4:20, Wed 2:30–4:20
PHONE: (617) 627-3262

PREREQUISITES: High-school algebra

TEXT: C. C. Chen and K.-M. Koh, *Principles and Techniques in Combinatorics*, World Scientific, Singapore, 1992. ISBN 978-981-02-1139-4.

COURSE DESCRIPTION: Combinatorics is the mathematics of counting. Since many problems in science and in mathematics require counting, combinatorics has a wide range of applications. A few areas of mathematics, including number theory, probability, and graph theory, in particular rely heavily on combinatorics.

In this course we will study the basic techniques and principles of combinatorics. Starting with the very simple addition and multiplication principles, we study permutations, combinations, circular permutations, and arrangements and selections with repetitions. After that we move to multinomial coefficients and Pascal's triangle, the pigeonhole principle, Ramsey numbers, the principle of inclusion and exclusion, the Sieve of Erathosthenes and Euler φ-function. Finally, we introduce the powerful techniques of generating functions and recurrence relations.

The goal of the course is to teach enough basic techniques and principles that a student can solve a problem such as the following:

Example. *In a hotly contested election year, each senator slaps the face of one other senator (just one). Observing senatorial decorum, no two senators slap each other. Irrespective of how the senators slap one another, is it always possible to form a committee of 35 senators none of whom has slapped another committee member?*

This course is highly recommended to students interested in the Putnam Competition; many problems in the Competition have traditionally been in combinatorics.

Math 61 | Discrete Mathematics | Fall2018

Course Information

BLOCK: F+, T,Th, 12noon -1:15 AM
INSTRUCTOR: Montserrat Teixidor i Bigas
EMAIL: mteixidor@tufts.edu
OFFICE: Bromfield-Pearson 115
OFFICE HOURS: (Spring 2018) by appointment
PHONE: 7-2358

PREREQUISITES: Math 32, Comp 11 or consent.

TEXT: to be decided

COURSE DESCRIPTION:

This course is an introduction to discrete mathematics. In mathematics, discrete" is a counterpart to continuous", describing things that vary in jumps rather than smoothly. Calculus studies (mostly) continuous phenomena; the mathematics of the discrete largely fit into the area called combinatorics,where the core questions have to do with counting or enumeration. Our topics will include sets and permutations, equivalence relations, graph theory, propositional logic, cardinality, and more if time allows.

While the tools of discrete mathematics are simple in nature, they allow to deal with particular cases of basic profound problems in a variety of areas from Topology to Applied Mathematics and from Probability to Computer Science. Therefore, this course gives a glimpse of what may be ahead to those students that would like to consider taking more advanced Math classes in the future.

This course is intended as a bridge from the more computationally-oriented classes like calculus to upper-level courses in mathematics and computer science. Learning how to read and write proofs is an important component of this class and you will get a lot of help in the process.

Math 61, which is co-listed as Comp 61, is required for computer science majors. It counts toward the math major and is highly recommended for math majors (or possible math majors) who want a first glimpse of higher mathematics before taking 100 level courses.

Your grade in the course will be based on homework assignments, quizzes, two midterm exams and a final

Math 70 **Linear Algebra** **Fall 2018**

Course Information

Prerequisites: Math 34 or 39, or consent of instructor.

Text: Linear Algebra and Its Applications , 4th edition, by David Lay, Addison-Wesley (Pearson), 2011.

Course description: Linear algebra begins the study of systems of linear equations in several unknowns. The study of linear equations quickly leads to the important concepts of vector spaces, dimension, linear transformations, eigenvectors and eigenvalues. These abstract ideas enable efficient study of linear equations, and more importantly, they fit together in a beautiful setting that provides a deeper understanding of these ideas.
Linear algebra arises everywhere in mathematics. It plays an important role in almost every upper level math course. It is also crucial in physics, chemistry, economics, biology, and a range of other fields. Even when a problem involves nonlinear equations, as is often the case in applications, linear systems still play a central role, since “linearizing” is a common approach to non-linear problems.
This course introduces students to axiomatic mathematics and proofs as well as fundamental mathematical ideas. Mathematics majors and minors are required to take linear algebra (Math 70 or Math 72) and are urged to take it as early as possible, as it is a prerequisite for many upper- level mathematics courses. The course is also useful to majors in computer science, economics, engineering, and the natural and social sciences.
The course will have two midterms and a final, as well as daily homework assignments.

Math 87 Mathematical Modeling and Computation Fall 2018

Course Information

Block: D+TTh, 10:30 -11:45 AM
Instructor: Caleb Magruder
Email: `caleb.magruder@tufts.edu`
Office: Bromfield-Pearson 105
Office hours: (Fall 2018) T&Th 12:30-2:00 PM
Prerequisites: Math 34 or 39, Math 70 or 72, or consent.

Text: none

Course description:
This course is about using elementary mathematics and computing to solve practical problems. Single-variable calculus is a prerequisite, as well as some basic linear algebra skills; other mathematical and computational tools, such as elementary probability, elementary combinatorics, and computing in MATLAB, will be introduced as they come up.

Mathematical modeling is an important area of study, where we consider how mathematics can be used to model and solve problems in the real world. This class will be driven by studying real-world questions where mathematical models can be used as part of the decision-making process. Along the way, we'll discover that many of these questions are best answered by combining mathematical intuition with some computational experiments.

Some problems that we will study in this class include:

1. The managed use of natural resources. Consider a population of fish that has a natural growth rate, which is decreased by a certain amount of harvesting. How much harvesting should be allowed each year in order to maintain a sustainable population?

2. The optimal use of labor. Suppose you run a construction company that has fixed numbers of tradespeople, such as carpenters and plumbers. How should you decide what to build to maximize your annual profits? What should you be willing to pay to increase your labor force?

3. Spring networks. Structural modeling of mechanical networks can be used to estimate the displacement of bridges and buildings. How much does a bridge bend when loaded with vehicles? Can structural modeling anticipate resonant modes of vibration in a network? Is it possible to detect structural defects of a bridge from displacement measurements alone?

Using basic mathematics and calculus, we will address some of these issues and others, such as dealing with the MBTA T system and penguins (separately of course...). This course will also serve as a good starting point for those interested in participating in the Mathematical Contest in Modeling each Spring.

Math 121 / Bio 121 **Mathematical Neuroscience** **Fall 2018**

Course Information

Block: C (Tu, We, Fr 9:30 –10:20)
Instructor: Christoph Börgers
Email: `cborgers@tufts.edu`
Office: Bromfield-Pearson 215
Office hours: (Spring 2018) Tu, We 4:30–6:00
Phone: (617) 627-2366

Prerequisites: Math 51, and willingness to do Matlab programming. (Programming experience is desirable but not necessary. Programming will involve modification of Matlab code given to you.) No biological background will be assumed.

Text: Christoph Börgers, An Introduction to Modeling Neuronal Dynamics, Springer 2017. Sells on Amazon for $54.25 (hardcover). You can also just download a pdf of it for free from Tisch Library.

Course description: This is a course on differential equations in neuroscience, a field that began with a series of papers by the physiologists A. L. Hodgkin and A. F. Huxley in the *Journal of Physiology* in 1952. At the time it had been well known for a long time that the interior voltage of a nerve cell can spike, and that voltage spikes are the basis of neuronal communication. Hodgkin and Huxley explained the mechanism underlying the spikes, and reproduced it in a system of differential equations modeling the nerve cell. These equations are now called the Hodgkin-Huxley equations. Eleven years after their groundbreaking series of papers appeared, Hodgkin and Huxley were awarded the Nobel Prize.

Theoretical neuroscience can be viewed as the study of coupled systems of Hodgkin-Huxley equations. The goal is to understand the human brain. We are very far from reaching this goal, but a lot has been understood about the transitions from rest to spiking in various different types of neurons, the propagation of signals through neuronal networks, synchronization of activity in large groups of neurons, origins and functions of oscillatory activity in neuronal networks (as observed for instance in the EEG), and other issues related to the physics of the brain. We will study some of the known mathematical results, and think about what they might suggest about brain function and disease.

The two main sets of tools used for studying differential equations modeling individual neurons and neuronal networks are qualitative methods for the study of differential equations, and computational simulation. You will see examples of both in this course. Even if you do not plan to study neuroscience in the future, this course can serve as an introduction to using ordinary differential equations for the purpose of thinking about science.

Math 123 **Data Analysis** **Fall 2018**

Course Information

PREREQUISITES: Math 42, Math 70/72, and the ability/willingness to program in Matlab or Python.

TEXT: None. I will distribute the slides that augment lectures.

COURSE DESCRIPTION: This class is about the computation analysis of large sets of high-dimensional data, thought of as large clouds of points in R^d. We focus on fundamental ideas based on Linear Algebra, presented without assuming a background in Statistics. Topics include:

1. Data embeddings, including principal component analysis, a tool for representing high-dimensional data in lower-dimensional linear spaces.
2. Unsupervised learning algorithms including k-means clustering and spectral clustering
3. Support vector machines, a class of methods for classifying points into two ore more categories based on training examples. Approaches for constructing models are primarily based on optimization algorithms, though no background in optimization beyond multivariable calculus is assumed. We will also discuss the connection between multi-class classification and mutli-candidate elections.
4. Most algorithms in this course have a linear flavor: PCA reduces to a lower-dimensional linear space; clustering algorithms and support vector machines separate clusters by hyperplanes. However, by first non-linearly mapping the data into a different space, then applying the "linear" methods on the images, one can use PCA to reduce data more generally to lower-dimensional surfaces, or separate clouds of points using hypersurfaces. There is a subtlety here, known as the kernel trick, which allows doing the computation without evaluating the nonlinear map too often, or even without explicitly specifying it at all.

This course is suitable for upper level undergraduate and beginning graduate students from Mathematics, Engineering, Computer Science, or those with particular interest in data analysis and pattern classification. **This course will consist of lectures and programming exercises.**

MA126 / CS126 Numerical Analysis Fall 2018

Course Information

Block: G+ (MW 1:30-2:45)
Instructor: Bruce Boghosian
Email: `bruce.boghosian@tufts.edu`
Office: Bromfield-Pearson 211
Office hours: TBA
Prerequisites: MA51 or MA 155 and programming ability in, e.g., C, C++, Fortran, Matlab, or Mathematica.
Text: TBA

Course description:

At this point in your mathematical career, you have developed some intuition for real numbers and their properties. You have spent some time studying real-valued functions of a real variable, including such properties as continuity and differentiability, as well as real-valued solutions to dynamical equations of importance in mathematics, science and engineering.

Real numbers are of such fundamental importance in mathematics that it may surprise you to learn that digital computers can not represent them in any exact sense. Instead, digital computers approximate real numbers by so-called floating-point numbers. The properties of floating-point numbers and functions thereof are sufficiently different from those of real numbers, that a separate branch of analysis is needed to study them, namely *numerical analysis*. More generally, numerical analysis is the study of algorithms for mathematical problems involving real or complex numbers, such as differentiation and integration, linear systems of equations, nonlinear algebraic equations, and differential equations.

While the approximation of real numbers by floating-point numbers is very effective for most applications, it is far from perfect. As an example, consider the following simple *dynamical system* on the set of real numbers $[0, 1)$: One step of the evolution consists of doubling the number and then taking the result modulo one [1]. It is easy to see that this dynamical system has closed orbits. For example, in one step of this evolution $1/3$ maps to $2/3$, and in the next step $\frac{2}{3}$ maps back to $1/3$, so $1/3 \rightleftarrows 2/3$ is a closed orbit of period two. There is only one problem with this example: It won't work if implemented on a computer using floating-point arithmetic. You can try starting with the closest floating-point number to $1/3$, but no matter what you do, you will not return to that number in two iterations. The orbits of the real-valued dynamical system are very different from those of its floating-point-valued representation. Moreover, no matter where you start in the unit interval, after 52 iterations of the above algorithm you will find that the result is zero forever thereafter!

In spite of such worrisome issues, numerical algorithms involving floating-point numbers are widely used in since and engineering, and their importance is arguably increasing in time. They are used to predict the airflow and resultant drag of airflow over an automobile. They are used to simulate the motion of molecules in chemical reactions. They are used to determine the best web pages for you to examine after you have typed search terms into Google. It has been estimated that the amount of computer cycles saved by the development and adoption of clever numerical algorithms during the twentieth century was comparable to the amount of time saved by increasing processor speeds.

This course is an introduction to numerical analysis, treating linear algebra fairly lightly, instead emphasizing non-linear equations, integration, and differential equations. (For a thorough treatment of numerical linear algebra, take Math 128/CS 128.) Computer programming will be a substantial component of the homework.

[1] In other words, double the number. If the result is bigger than one, subtract one from it. This is sometimes called the *Bernoulli map*.

Math 135 | Real Analysis I | Fall 2018

Course Information

BLOCK:
INSTRUCTOR: Alex Hening
EMAIL:
OFFICE:
OFFICE HOURS:
PHONE:

PREREQUISITES:: Math 42 or 44, and 70 or 72, or consent.

TEXT:: Rudin, Principles of Mathematical Analysis, 3rd edition. We will cover chapters 1-5.

COURSE DESCRIPTION: This course will be about the study of real functions, their derivatives and integrals. It is a foundational course for mathematics, both pure and applied. While calculus is more about intuition and computation, the emphasis in real analysis is on rigorous proofs. Our intuition does not always give the correct answer. That is why rigorous proofs are required in mathematics. Some of the mathematical concepts are not immediately easy to grasp, and our intuition can lead us astray. For example the set $\mathbb{Z} = \{\dots, 2, 1, 0, 1, 2, 3, \dots\}$ of integers is a proper subset of the set of rationals $\mathbb{Q} = \{\frac{m}{n} \mid n \neq 0, m, n \in \mathbb{Z}\}$. However, as we will show during the course, there is a one-to-one correspondence (bijection) between $\mathbb{Z}$ and $\mathbb{Q}$. In this sense, the infinite sets $\mathbb{Z}$ and $\mathbb{Q}$ have the same number of elements. However, we will show there is no one-to-one correspondence between the real numbers $\mathbb{R}$ and the rational numbers $\mathbb{Q}$ - even though both sets have an infinite number of elements the set of real numbers is larger. We will study metric spaces, compactness, connectedness, continuous mappings and modes of convergence. We will also encounter results from calculus, such as the intermediate value theorem, but in a more general setting that enlarges their applicability. By studying the fundamental ideas in a general setting, we will gain a deeper understanding of the ideas and concepts involved. The primary goal of this course is introducing the main concepts from real analysis. A secondary goal is to get students used to reading and writing rigorous proofs. There will be weekly homework, one midterm exam, and a final (and possibly a few quizzes).

Math 145 | Abstract Algebra Course Information | Fall 2018

BLOCK: I+ (Mo, Wed 15:00 – 16:15), K+ (Mo, Wed 16:30 – 17:45)
INSTRUCTOR: Robert Kropholler
EMAIL: robert.kropholler@tufts.edu
OFFICE: SEC 010
OFFICE HOURS: (Spring 2018) Mon, Wed 12:00 – 13:00
PHONE: (617) 627-2363

PREREQUISITES: Linear Algebra (Math 70 or 72).

TEXT: TBD

COURSE DESCRIPTION:

Algebra is one of the main branches of mathematics. Historically, algebra was concerned with the manipulation of equations and, in particular, with the problem of finding the roots of polynomials. This is the algebra that you know from high school.

There are clay tablets from 1700B.C. showing that the Babylonians knew how to solve quadratic equations. Formulas for the roots of cubic and fourth degree polynomial equations were found in Italy during the Renaissance. About the time of Beethoven, however, a young French mathematician named Evariste Galois made the dramatic discovery that for polynomials of degree greater than four, no such formula exists. To do this, Galois had to introduce a completely new branch of mathematics which known as group theory.

The concept of a group is now one of the central unifying themes in mathematics. Roughly speaking, group theory is the study of symmetry. In mathematics, there are deep connections between group theory, geometry and number theory. Group theory also plays a central role in the efforts of physicists to describe the basic laws of nature.

The principle (but by no means the only) goal of Math 145-146 is to describe the connection between group theory and the roots of polynomials. Ring theory is the natural setting for the study of polynomials. In Math 145, we will introduce groups and rings and investigate their basic properties.

MATH 0150-01 Michael Chou
Fall 2018

Ocvjgocvkecn Etarvqitcrja (Mathematical Cryptography)

BLOCK: G (MWF 1:30-2:20)
INSTRUCTOR: Michael Chou
EMAIL: Michael.Chou@tufts.edu
OFFICE: 107 Bromfield-Pearson Hall
OFFICE HOURS: (Spring 2018) TBD
PHONE: (617) 627 2678
PREREQUISITES: Math 70. Familiarity with proofs (e.g. Math 61, 63, 72, 135, or 145).

TEXT: TBD

COURSE DESCRIPTION: Cryptography has become an increasingly hot topic especially as the production of reasonable scale quantum computers threatens to undermine our current encryption schemes. In this course we will cover the basics behind modern day encryption: Diffie-Hellman key exchange, RSA encryption, and investigate the mathematics behind these ideas. We will explore mathematical attacks to various encryption schemes, and discuss why the advent of quantum computers threatens to compromise these ideas. Further, we will introduce a budding tool: elliptic curve cryptography.

We will introduce these mathematical objects and discuss how they can be implemented in cryptography. Finally, we hope to discuss lattice based cryptography, another alternative that is (so far) safe in the face of quantum computing. If time permits, we will investigate cryptocurrencies and the associated mathematics.

Grades will be determined by weekly homework assignments, a take-home midterm, and a final presentation or project, depending on enrollment. Assignments will include primarily proof based questions, so a good foundation in proof writing will be necessary. No programming knowledge will be needed.

Math 150-02 | Numerical Optimization | Fall 2018

Course Information

BLOCK: F+ (Tu, Th 12:00pm –1:15pm)
INSTRUCTOR: Xiaozhe Hu
EMAIL: Xiaozhe.Hu@tufts.edu
OFFICE: 212 Bromfield-Pearson Hall
OFFICE HOURS: (Spring 2018) by appointment
PHONE: (617) 627-2356

PREREQUISITES: Math 126 and 128 or consent. Some programming ability in a language such as C, C++, Fortran, Matlab, etc.

TEXT: TBD

COURSE DESCRIPTION:

Many problems in science, engineering, economics, and industry involve optimization, in which we seek to optimize objective function subject to constraints. Due to the wide and growing use of optimization in different areas, such as model fitting, machine learning, optimal control, and image processing, it is important for practitioners to understand optimization algorithms and their impact on various applications.

This course is an introductory survey of optimization algorithms and gives a comprehensive description of the state-of-the-art techniques for solving optimization problems. The primary objective of the course is to develop an understanding of optimization algorithm and, more importantly, the capabilities and limitations of these algorithms. We will focus on both theoretical foundations (e.g., convergence analysis, error analysis) and practical implementations (e.g., computational complexity, computer storage). The emphasis will be the accuracy, numerical stability, efficiency, robustness, and scalability. The topics include: gradient descent method, (nonlinear) conjugate gradient method, (quasi/inexact) Newton's method, trust region method, least-squares, incremental and stochastic gradient descent, (randomized) coordinate descent, acceleration techniques, and stochastic optimization. In-class examples, homework, and projects will feature some of the practical applications of these algorithms to solve real-world problems. Homework problems will consist of written problems as well as computer programming assignments in Matlab (or your favorite programming language).

This course is suitable for upper level undergraduate and beginning graduate students from Mathematics, Engineering, Operations research, Computer science, Biology, Chemistry, and Physics.

Math 155 **Nonlinear Dynamics and Chaos** **Fall 2018**

Course Information

Time: Block L+, TTh 4:30–5:45pm
Instructor: Eunice Kim
Office: SEC LL 010
Office hours: (Spring 2018) T 10:30–11:20am Th 2:00–3:00pm
Phone: 617-627-6308
e-mail: eunice.kim@tufts.edu

Prerequisites: One of the following two: Math 42 or Math 44, and one of the following three: Math 51 or Math 70 or Math 72.

Text: Steven Strogatz, Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering, 2nd edition, Westview Press.

This is a course on ordinary differential equations, with emphasis on qualitative, geometric aspects of the subject. Main mathematical ideas will be motivated and illustrated extensively using applications such as population growth models, computational epidemiology and thresholds in epidemics, the competitive exclusion principle in ecology, predator-prey cycles, planetary motion, synchronization of oscillators, pulsatile drug delivery using an oscillating chemical reaction, chaos and coding.

Content: The mathematical ideas studied in the course are:

1. equilibria and their stability
2. limit cycles and their stability
3. saddle-node, pitchfork, transcritical, Hopf, and homoclinic bifurcations, and structural stability of bifurcations
4. flows and their reduction to iterated maps
5. chaotic dynamics, strange attractors, fractal dimension

The course is suitable for upper level undergraduate and beginning graduate students from Mathematics, Engineering, Biology, Chemistry, and Physics.

Applied Mathematics majors can use Math 155 in place of Math 51 to satisfy their Differential Equations requirement.

Math 161 | Probability | Fall 2018

Course Information

BLOCK:
INSTRUCTOR: Alex Hening
EMAIL:
OFFICE:
OFFICE HOURS:
PHONE:

PREREQUISITES:: Math 42 or consent.

TEXT::

COURSE DESCRIPTION: The theory of probability was originally developed in the 17th century by Blaise Pascal and Pierre de Fermat in order to understand gambling. Today, probability has found many applications in science and engineering:

- Ecologists use probability in order to predict the likelihood of extinction or survival of certain species.
- Economists observe the state of the economy over periods of time and use the information to forecast economic events.
- Insurance companies use probability for risk assessment and modeling.
- Governments use probabilistic methods for environmental and financial regulation.

Probability will help you understand 'paradoxes' like the Monty Hall problem:

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

Answer: **Yes**, if you switch your chance of winning is 2/3, while if you do not switch your chance of winning is only 1/3.

In this course, you will learn the basic terminology and concepts of probability theory, including random experiments, sample spaces, probability distributions, probability density functions, and expected values.

There will be weekly homework, one midterm exam, and a final (and possibly a few quizzes).

Math 167 — Differential Geometry — Fall 2018
Course Information

BLOCK: M+ (Mon, Wed, 6:00–7:15 p.m.)
INSTRUCTOR: Loring Tu
EMAIL: `loring.tu@tufts.edu`
OFFICE: Bromfield-Pearson 206
OFFICE HOURS: (Spring 2018) Mon 2:50–4:20, Wed 2:30–4:20
PHONE: (617) 627-3262

PREREQUISITES: Math 70 and 135

TEXT: To be announced.

COURSE DESCRIPTION: Topology and geometry are both concerned with properties of spaces and figures in them. Broadly speaking, topology deals with "shape", while geometry deals in addition with "size". Differential geometry studies curves, surfaces, and their higher-dimensional analogues using techniques of differential calculus. It serves to unify various types of geometries – Euclidean, spherical, hyperbolic, and projective geometries. Developed largely in the nineteenth century, differential geometry was inspired in part by the discovery of non-Euclidean geometry and in part by problems in optics, mechanics, astronomy, and geodesy. In the twentieth century, differential geometry laid the mathematical foundation for the general theory of relativity and remains an active branch of modern mathematics.

The emphasis of this course is on differential-geometric concepts that are valid in all dimensions, although for pedagogical reasons we study their manifestations mainly for curves and surfaces. The topics to be covered are the local and global theory of curves and surfaces, curvature, parallel translation, geodesics, and vector fields. We will be asking questions like: How does one measure the curvature of curves and surfaces? How does the curvature determine the topology? What is the analogue of a straight line on a curved surface? How does non-Euclidean geometry resolve the problem of the consistency of Euclid's parallel postulate? The course culminates in the Gauss-Bonnet theorem, which gives a surprising link between geometry and topology.

This course has several goals:

(i) to introduce the students to differential geometry and to such classical results as models for non-Euclidean geometry, the the Gauss-Bonnet theorem, and the index theorem for vector fields, which lie at the core of our mathematical heritage;

(ii) through extensive application of vector calculus and linear algebra, to reinforce what the students have learned in earlier courses;

(iii) as complement to real analysis, since many of the concepts of real analysis, such as continuity, differentiability, compactness, and connectedness, appear here in a more geometric and intuitive setting.

Math 211 **Analysis** **Fall 2018**

Course Information

BLOCK: D+ (Tue Thu 10:30 – 11:45 a.m.)
INSTRUCTOR: Fulton Gonzalez
EMAIL: `fulton.gonzalez@tufts.edu`
OFFICE: Bromfield-Pearson 203
OFFICE HOURS: (Spring 2018) Mondays, 5:00 –6:00 p.m., 7:30 –8:30 p.m., and Wednesdays 7:30 – 8:30 p.m.
PHONE: 617 627 2368

PREREQUISITES: Math 135 or equivalent, Math 136 is recommended.

TEXT: *Real and Complex Analysis*, 3th Edition, by Walter Rudin, 3rd. ed. (1987), McGraw-Hill.

COURSE DESCRIPTION: Define the function $f(x)$ on the interval $[0,1]$ by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational,} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

It is easy to see that the Riemann integral[1] of $f(x)$, defined as the limit of Riemann sums, does not exist. However, since almost all; i.e., all except a countable number of, real numbers are irrational, one would, in a sense, want the integral of $f(x)$ to exist and be equal to zero. Lebesgue's theory of integration, which first appeared in his famous 1904 book [*Lecons sur integration et la recherche des fonctions primitives*, Gauthier-Villars, Paris, 1904; second edition, 1928] treats discontinuous functions as "natural" objects to integrate and paves the way for integration on spaces besides Euclidean space (e.g. topological groups). It is immediate from Lebesgue's construction that the integral of $f(x)$ exists and equals zero.

In this course, we will introduce measure theory and the tools needed to define abstract integrals, and in particular the Lebesgue integral on $\mathbb{R}^n$. We will explore important concepts such as the great convergence theorems (i.e., under what conditions is $\lim \int f_n = \int \lim f_n$?), L^p spaces, Banach and Hilbert spaces, complex measures, and various ways of differentiating measures, including the Radon-Nikodym Theorem. Finally, we study Fourier transforms and some applications. The course will roughly follow the first eight chapters of Rudin's book.

There will be two exams - a midterm and a final - as well as weekly problem sets.

[1] Which was actually formally defined by Cauchy.

Math 219 | Algebraic Topology | Fall 2018

Course Information

BLOCK: R+MW, Mon Wed 9-10:15
INSTRUCTOR: Genevieve Walsh
EMAIL: `genevieve.walsh@tufts.edu`
OFFICE: SEC LL011
OFFICE HOURS: please email
PREREQUISITES: Math 135 and 145 or equivalent, 168 recommended

TEXT: *Algebraic Topology* by Allen Hatcher. Cambridge University Press 2002. Available at https://www.math.cornell.edu/ hatcher/AT/ATpage.html

COURSE DESCRIPTION:

Algebraic Topology is the study of algebraic invariants associated to topological spaces. We will discuss the fundamental group, homology and cohomology, as well as some relations between these invariants.

This course will approach this study from a de- cidedly geometric viewpoint. We will begin by reviewing some underlying geometric notions, such as homotopy. We will then define the fundamental group, using Van Kampen?s theorem as our main tool for computations. We will also use the fundamental group to understand covering spaces, and define a correspondence between subgroups of the fundamental group of a space and covers of that space. Next we will turn to an abelian theory, homology. Although somewhat more complicated to define, this is an extremely useful tool. This theory assigns a sequence of abelian groups to a space, called the homology groups. The first of these groups is the abelianization of the fundamental group. Homology groups can be computed naturally using a cell complex. Finally, we will study cohomology and Poincare duality for manifolds. Throughout, examples and geometric constructions will be emphasized, with a particular emphasis on 2- and 3-dimensional manifolds, graphs, and 2-dimensional complexes.

There will be a midterm and a final, and biweekly problem sets. Participants in the class will occasionally present solutions. This course icontains some preparation for the qualifying exam in Algebraic Topology.

Math 250 **Harmonic Analysis** **Fall 2018**

Course Information

Block: N+ (Tue Thu 6:00 – 7:15 p.m.)
Instructor: Fulton Gonzalez
Email: `fulton.gonzalez@tufts.edu`
Office: Bromfield-Pearson 203
Office hours: (Spring 2018) Mondays, 5:00 –6:00 p.m., 7:30 –8:30 p.m., and Wednesdays 7:30 – 8:30 p.m.
Phone: 617 627 2368

Prerequisites: Math 211 or instructor permission.

Texts: *Unitary Representations and Harmonic Analysis*, by Mitsuo Sugiura, North-Holland (1990). This book is available online, from a Tufts IP address, via Science Direct.

Abstract Harmonic Analysis, by Gerald Folland, CRC Press (1995).

Course description: Any "reasonable" function f on the unit circle T can be expanded in a Fourier series

$$f(e^{i\theta}) = \sum_{n=-\infty}^{\infty} a_n\, e^{in\theta}.$$

The fact that T is a compact group plays an essential role in this expansion: the one-dimensional function spaces $\mathbb{C}\, e^{in\theta}$ are precisely the irreducible unitary representation spaces of $T = \mathbb{R}/\mathbb{Z}$.

We will generalize the theory of Fourier series on T to the L^2 decomposition of any compact (including any finite) group, via its irreducible unitary representations. We will also study the unitary representations and L^2 theory of certain classes of noncompact groups, with $\mathrm{SL}(2,\mathbb{C})$, $\mathrm{sl}(2,\mathbb{R})$, and the Heisenberg group as our paradigms.

Along the way, some essential mathematics will be introduced. These will include aspects of C^* algebras and the spectral theory of bounded linear operators, Haar measures, induced representations, and character theory. We'll also develop some essential aspects of Lie theory, such as Iwasawa decompositions and highest weight representations . Finally, we'll consider some applications to integral geometry.